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ECONOMIC ANALYSIS OF CROPPING SYSTEMS IN DRYLAND FARMING

FINAL REPORT

Old West Regional Commission Project Number 10570032
(cooperatively with the Montana Agricultural Experiment Station)

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I. SUMMARY AND PRACTICAL INTERPRETATIONS

The results of this research did not completely achieve the original objective of estimating optimal cropping strategies for dryland farming in the Northern Great Plains, but a good deal of progress was made toward this ambitious objective. Failures at various locations were caused by inadequacies of available data; usually deficiencies in the quality of soil moisture measurements in the experiments were the problem. But during the last two decades there has been an apparent reluctance to conduct controlled experiments over an extended period of time at a single location. The research reported here does demonstrate that it is possible to derive economically optimal estimates of cropping strategies when reasonably adequate data are available, experimental data which is very feasible to obtain with a consistent effort directed to that purpose.

It would appear that quite reliable decision rules were obtained for the Highwood Bench-Fort Benton Area of Montana from only seven years of experimental data. The only serious weak link in the model developed for these data was the estimated probability distributions for plant available soil water after crops of winter wheat and barley. This weakness is unavoidable with only seven years of data and can be made more precise by only two methods: (1) extension of the experiment over additional years which is more costly in terms of the delay in availability of the information; and, (2) devote a considerable research effort to synthesis of plant available soil moisture from year to year using climatic data and evapotranspiration formulae. The latter is no simple matter if a useful level of accuracy is to be achieved.

The economic analysis on the Highwood data showed that the alternate winter wheat-fallow rotation is an excellent cropping system _if_ saline seep damages to the land are not a problem and _if_ disease and weed problems can be kept under control. But we know that saline seep is a serious problem in this region and it would be irresponsible to ignore land losses in order to receive modestly higher annual cash flows.

It must be recognized that experimental yields do not always reflect all of the disease and weed hazards associated with various crop rotations because sites are deliberately chosen to avoid such problems in order to prevent the confounding of these problems with controlled factors of the experiment. Consequently, we would expect yields from the alternate winter wheat-fallow rotation in the experimental data to be overly optimistic relative to rotations which included both fallow and barley with winter wheat in a rotation. Bringing barley into the rotation would reduce the disease and weed hazards actually faced on the typical wheat farm, as contrasted to the selected experimental site. In view of this practical, subjective element in translating experimental results to on-farm situations, the economic analysis for the Highwood data was performed in such a way that the individual farmer can make the subjective evaluation for his particular conditions, possibly with advice from agronomists or plant pathologists.

The methodology used in this prime winter wheat region of Montana was to estimate the expected opportunity loss in net cash returns from using various probabilistic strategies which would tend to control saline seep. Then these expected opportunity losses in dollars per acre were crudely translated into increased yield in winter wheat production required to compensate for expected opportunity losses for a given strategy.

One of the most promising strategies was to recrop winter wheat stubble with barley in the spring if plant available soil water is greater than 5.0 inches and to recrop barley stubble in the fall with winter wheat if plant available soil water exceeds 2.0 inches. When one of these critical soil water levels is not met, the land is fallowed, and winter wheat is always planted after a year of summer fallow. Average land use frequencies in this strategy are approximately 25 percent fallow, 46 percent winter wheat, and 29 percent barley. Fallow is chosen in this strategy only when the soil is quite dry at the time a decision must be made, which gives good control of deep water percolation, at least on the average. Control of deep water percolation provides a control for saline seep.

It was shown that less than a bushel per acre increase in yield of winter wheat, which can be attributed to getting barley into the cropping system with a relative frequency of about 30 percent, will justify this cropping strategy over alternate winter wheat-fallow. However, this assertion does not take into consideration differences in fixed costs under the two different systems. These fixed costs and any other problems such as labor requirements associated with the change from winter wheat-fallow are quite specific to individual farms. Some farmers may already have slack in their machinery and labor supplies so that only variable costs are relevant, while others may be in a very tight situation.

Using the budget data for a typical farm in Chouteau County (Schaefer, et al., 1976c), fixed costs for machinery would be about $2 per acre more under the above described strategy than under alternate wheat-fallow. This measure of the difference was obtained by calculating the weighted average of fixed costs in machinery for land in fallow and in crop for each of the two cropping systems; the weights for alternate wheat-fallow were 50-50, while for the strategy based on soil water they were 25-75 for fallow and crop, respectively. This approach has some deficiencies in that the allocation of fixed costs to fallow and wheat land is somewhat arbitrary in a budget study such as in (Schaefer, et al., 1976c), but the estimate would be expected to be reasonable.

A large component of fixed costs is in harvesting equipment. It would appear to be economic for a farmer following a strategy based on soil water to rely partially on custom harvesting during unusual years in which a larger proportion of his farm was in crop. A strategy which gives a relative frequency of crop equal to 75 percent will involve years with substantially more or less than 75 percent of the farm in crop because of the statistical nature of the decision rule based on soil water at seeding time.

Fluctuations in machinery and labor requirements can be substantially reduced by keeping the farm divided into strips, much as in the old crop-fallow system, and introducing more complicated strategies which deliberately break up the tendency for the various strips to be in crop or fallow at the same time. The general idea is to use different critical soil water levels in the decision rules for two separate tracts of land when both tracts are in the same land use (barley or winter wheat stubble). This type of interrelated strategy for the two tracts will put one tract in fallow and the other in crop with a probability equal to that of soil water falling between the critical soil water levels used on the separate tracts. More detail on these general strategies is given in (Burt and Stauber).

Plant available water in the soil is not often measured by farmers and would entail considerable cost if a good sampling procedure were used. As a practical approximation, depth of wet soil calibrated to the soil texture has been suggested. Another advantage of using depth of wet soil in place of plant available water is that this substitution can be used as a method for extrapolating the results of the analysis to other soils than the experimental site.

We would expect the qualitative relationship between the optimal critical soil water level in a decision rule and the water holding capacity of the soil to be directly related, ceteris partibus. That is to say, a higher soil water capacity within a given depth range would imply that the best choice of the critical level for soil water, above which a crop is planted and below which the land is fallowed, would also be higher and vice versa. By a higher soil water capacity, we mean relative to the experimental site.

In view of this direct relationship, a proportional adjustment between the critical level of soil water and field capacity of the soil would seem to make sense as an approximate adjustment for soils not too different from the experimental site. If depth of wet soil is substituted for plant available water, calibrated to correspond to the soil at the experimental site and the critical level of a decision rule, then a definition of the strategy in units of wet soil at seeding time will automatically make the proportionate adjustment when the strategy is applied to other soils.

Plant available soil water capacity at the Highwood Bench experiment is approximately 12.4 and 10.0 inches, in the top 5 and 4 feet, respectively. The strategy using 2.0 and 5.0 inches of water as the critical values in the fall and spring would transform into 10 and 24 inches of wet soil in the fall and spring, respectively. The fall decision for winter wheat considers soil water to 5 feet, while in the spring, only the top 4 feet of soil water is relevant for barley.

In summary, the following strategy is being proposed for dryland grain farming in the Highwood-Fort Benton area to mitigate saline seep problems with a minimum of financial sacrifice:

1) Plant winter wheat on barley stubble if depth of wet soil measured from the surface is greater than 10 inches, otherwise fallow during the coming year.
2) Plant barley on winter wheat stubble if depth of wet soil measured from the surface is greater than 24 inches, otherwise fallow during the summer.
3) Plant winter wheat on land which has been summer fallowed regardless of soil water.

Average annual precipitation at the experimental site is about 15 inches. This same strategy would probably give good results in slightly drier areas in the region as long as average annual precipitation is not less than 14 inches and winter wheat is still adapted to the climate. However, the expected opportunity loss would be higher under drier climates and the strategy should maybe be adjusted to somewhat greater depths of wet soil. For example in areas of only 13 inches average annual precipitation the critical levels 10 and 24 inches could logically be replaced by say 14 and 28 inches of wet soil, but we have no objective basis for such adjustments.

Results for the Spring Wheat Region of Northern Montana and North Dakota suggested that too much fallow is being practiced regardless of any saline seep problems which might be present. Under a simple long-run average net returns criterion, the results suggested that the very eastern part of Montana and all of North Dakota north of Mandan should probably be in continuous cropping, without any fallow except for cases where benefits from weed control can justify occasional summer fallow.

There was a large data gap between Eastern Montana and Havre in North Central Montana, so we have a large expanse of spring wheat country where nothing definitive can be said. The optimal strategy at Havre, using long-run average net returns in a simple model, was to plant wheat if plant available soil water is greater than about 3.5 inches and fallow otherwise. The critical level of soil water was between 3.0 and 4.0 inches depending on wheat price and yield assumptions. Data did not permit an analysis of other crops, such as barley, in the strategy.

The economic analysis showed that using 3.0 inches as the critical value resulted in a very small expected opportunity loss and this strategy appears to be quite effective for saline seep control. First of all, the land is in crop about two-thirds of the time under this strategy. But just as important, the land is fallowed only when soil water at seeding time in the spring is less than 3.0 inches. By contrast, the land is in fallow half of the time when alternate wheat-fallow is practiced, and during the years of fallow, soil water in the spring is greater than 3.0 inches 64 percent of the time. Both of these considerations make the strategy based on soil water much better from the point of view of saline seep control.

Transforming the strategy to inches of wet soil measured from the surface gives a critical level of 18 inches, which we propose as a practical strategy for the spring wheat region around Havre, Montana. Average annual precipitation for the Havre site is about 11.6 inches and average annual precipitation at Sidney, Montana where continuous cropping was optimal is about 14 inches. It would appear that fallow should start coming into the optimal strategy somewhere between 11.6 and 14 inches of average annual precipitation, but we are working with quite limited information and average annual precipitation, even at the same latitude, does not tell the entire story on climate.

Nevertheless, let us make two simple assumptions and estimate a "reasonable" strategy for the spring wheat region of Montana where the critical value for inches of wet soil at seeding time is correlated with average annual precipitation. First, we assume that fallow should start to enter the optimal strategy at about 14.0 inches average annual precipitation. Second, we assume that the critical value for inches of wet soil changes linearly with average annual precipitation.

These two assumptions give us a linear relationship between two variables and two points on that linear relationship, one point being associated with Havre and the other determined by our first assumption. The linear relationship implied by our assumptions is presented in tabular form in Table A. We even extrapolated to lower precipitation levels than Havre at the top of the table. Recall that the critical level for soil water at Havre was adjusted downward somewhat from the "optimal" level based on average annual net returns in order to improve saline seep control. Therefore, the strategies in Table A are relatively "dry" ones to help control saline seep.

A more "conservative" strategy in that it deviates less from the traditional rotation of alternate crop-fallow would be to use 4.0 inches of plant available water as the critical value at Havre. If this is done and the critical level transformed into inches of wet soil measured from the surface, the counterpart of Table A is Table B. If no saline seep problems are being experienced, or salts contamination of ground water supplies is no consideration, then the strategies implied by Table B are superior on economic grounds.

The critical levels given in Tables A and B are potential approximations for barley as well as spring wheat, although there are obvious risks in this kind of extrapolation. Since barley does not root quite as deeply as spring wheat, the critical levels might be adjusted downward by about 10 to 20 percent. One result of this study which casts considerable doubt on using Tables A and B for barley is the "pure fallow" effect on yield obtained for barley planted on fallow in the Highwood Bench experiment (see Section II). If this "pure fallow" effect is greater for barley than spring wheat, then a reduction in the critical levels of Tables A and B would be in the wrong

Table A

Critical Depths of Wet Soil for Spring Wheat Strategies

Average Annual Precipitation (inches)	Critical Depth of Wet Soil (inches)
11.00	22.5
11.25	20.6
11.50	18.8
11.75	16.9
12.00	15.0
12.25	13.3
12.50	11.3
12.75	9.4
13.00	7.5
13.25	5.6
13.50	3.8
13.75	1.9
14.00	0.0

Table B

Conservative Critical Depths of Wet Soil For Spring Wheat Strategies

Average Annual Precipitation (inches)	Critical Depth of Wet Soil (inches)
11.00	30.0
11.25	27.5
11.50	25.0
11.75	22.5
12.00	20.0
12.25	17.5
12.50	15.0
12.75	12.5
13.00	10.0
13.25	7.5
13.50	5.0
13.75	2.5
14.00	0.0

direction. Obviously, we are not in a position to say much about an optimal strategy for recropping with barley, more experimental data is needed.

One additional warning is in order for both the Spring Wheat Region and the winter wheat-barley strategies for North Central Montana. In both cases, empirical information was from open soils where there were no root restricting layers. Any root restricting layers will reduce the soil water reservoir available for storing water during a year of fallow. The practical versions of the strategies where the critical levels are expressed in inches of wet soil will partially adjust for this possibility, but the adjustments are much more appropriate for variations due to soil texture than for root restricting layers.

Most of the benefits of fallow emanate from increased water storage in the deeper layers of the soil, say between two and four feet from the surface, except in extremely dry years. If these lower depths for storage are unavailable, fallow would be beneficial only in exceptionally dry years. A good example of this condition is much of the Judith Basin in Central Montana where a gravelly layer restricts root penetration at about 24 inches. Experimental results show hardly any benefits from fallow which can be attributed to soil water storage in this area, but average annual precipitation exceeds 15 inches in most of the area too. On the basis of results from this study, we would expect fallow to be of questionable value for water storage in such a soil situation even with annual average precipitation as low as 13 inches.

A comparison of two maps of Montana, one showing saline seep problem areas and the other average annual precipitation contours, is quite suggestive that 13 to 14 inches average annual precipitation is a necessary condition for saline seep problems. The precipitation map had contours by two-inch intervals so that the comparison had to be rather crude, but a region of average annual precipitation with 14 or more inches can be identified for most of the major saline seep areas in the state.

The cropping strategy analysis suggested that on purely economic grounds, fallow should be practiced to a limited extent in the Spring Wheat Region when average annual precipitation exceeds 13 inches (see Tables A and B). Consequently, the saline seep problems in the Spring Wheat Region would seem to be largely solved by merely maximizing average profits without regard to the saline seep problems; clearly a case of no sacrifice to deal with saline seep. Fallow would be used only as needed to deal with weed problems or an occasional extra dry soil at planting time.

II. THE HIGHWOOD-FORT BENTON REGION OF MONTANA

This is one of the most productive agricultural areas of Montana, and also a major saline seep problem area. Climate and soils are excellent for winter wheat and other cereal grains. The predominant rotation since World War II has been alternate winter wheat-fallow. Considerable research effort has been devoted in recent years to analyze the saline seep problem. The concensus of opinion is that the primary source of water flow for the saline seeps emanates from deep percolation of water through the crop root zone during the summer fallow period.

The obvious solution to the problem is reduced summer fallow in the cropping systems followed in the area. The research being reported here was an effort to determine the economic incentives for continued summer fallow, and analyze alternative cropping strategies which would reduce deep water percolation.

The only useful empirical data available is a seven-year experiment which has been performed on the Highwood bench (1970-76). The experiment was organized and started in 1969 by Ray Choriki, and in recent years, has been conducted by Paul Brown with the Agricultural Research Service, USDA.

The data analyzed were from three rotations:

1. Barley-Winter Wheat
2. Fallow-Barley
3. Fallow-Winter Wheat

Each of these rotations was used in an experiment with nitrogen fertilizer as a controlled factor. Data from four levels of nitrogen were used, 0, 30, 60, and 90 pounds.

Plant available soil water was measured to five feet in the fall for purposes of winter wheat, and to four feet in the spring for use with barley. The silty clay loam has a relatively large storage capacity, about 12.4 and 10.0 inches of plant available water to five and four feet, respectively. Average annual precipitation for the area is a little under 15 inches.

The Statistical Model

A yield response surface for each winter wheat and barley was estimated, where the primary variables were total water available to the crop and nitrogen fertilizer. Total water was approximated by plant available soil water at seeding plus the sum of precipitation during May, June, and July. Nitrogen applied as fertilizer was used directly when the crop was planted on stubble, but when planted on fallow, a parameter was added to applied fertilizer nitrogen to account for extra nitrogen mineralized in the soil during the summer fallow year. This parameter was estimated statistically along with other parameters of the model, and is indirectly determined by yield response across nitrogen levels of the experimental data.

The following notation is introduced:

N = total nitrogen measured on a scale with origin (measure zero) at that level associated with no applied fertilizer for a crop planted on stubble (pounds per acre)

W = plant available soil water at seeding plus precipitation during May through July (inches)

$$X = \begin{cases} 0 \text{ if crop is planted on stubble} \\ 1 \text{ if crop is planted on fallow} \end{cases}$$

y = crop yield (bushels per acre)

The postulated functional form for the yield response surface is

$$(2.1) \quad y = b_0 + b_1N + b_2N^2 + b_3N^3 + b_4W + b_5W^2 + b_6W^3 + b_7NW + b_8N^2W + b_9NW^2 + b_{10}X,$$

N = (applied nitrogen) + a

The parameters to be estimated statistically are b_0, b_1, . . . b_{10} and a. The last parameter, a, is interpreted as the amount of nitrogen mineralized during the fallow year. The parameter b_{10} is a constant which is added to yield when the crop is planted on fallow (see the definition of X). The remainder of the yield equation is a general polynomial in N and W of third degree. The terms in NW, N^2W, and NW^2 are interactions which jointly reflect the nonadditivity of the net effects from nitrogen and water. The polynomial form for the response surface is justified by Taylor's formula of elementary calculus; any analytic function can be approximated by a truncated polynomial containing a sufficient number of terms.

Let the response function in (2.1) be written in abbreviated form as $f(N, W, X)$. Then the statistical specifications used to fit the equation to the data are

$$(2.2) \quad y_{jt} = f(N_{jt}, W_{jt}, X_{jt})v_{jt} \quad , \quad \begin{array}{l} j = 1, 2, \ldots 8 \\ t = 1, 2, \ldots 7 \end{array}$$

The subscript t is associated with a year of the experiment, and j designates a treatment combination, namely, a level of nitrogen fertilizer applied and the crop planted on either stubble or fallow. Thus, y_{jt} is observed crop yield, while N_{jt}, W_{jt}, and X_{jt} are the associated independent variables for a given observation.

The multiplicative variable v_{jt} is the statistical component of the model, an error term if you please. There are many sources and dimensions to this error term, such as measurement error in harvesting the plots, soil variation over plots, plant available water variation over plots, variation in stand, etc. All of these sources of error are random across treatments for a given year's data because of the randomized experimental design.

However, another source of error which is year-specific is variation in climatic conditions from year to year, other than the variation captured by the water variable W included in the equation. These other sources of variation are such things as temperature experienced before and during the growing season, the distribution of precipitation during the season, winter and early spring precipitation before May 1, disease and insect damage, relative humidity and wind, and probably many other things. The important aspect to recognize with regard to the year-specific component of the error term is that all treatments in a given year experience the same random shock.

In view of the above discussion, it makes sense to think of the error term as comprised of two factors, (2.3) $v_{jt} = \varepsilon_{jt} u_t$,

where ε_{jt} is a purely random factor associated with normal experimental unit variability in a given year, and u_t is a random shock imposed on yield from the many influences which are specific to a given year.

The multiplicative error term is justified on agronomic theory considerations as well as empirical results; the alternative would be an additive error.

The model of (2.2) is transformed into an additive error model by taking the natural logarithm of both sides of the equation to get

(2.4) $\log y_{jt} = \log (f(N_{jt}, W_{jt}, X_{jt})) + \log v_{jt}$

Using (2.3),

(2.5) $\log v_{jt} = \log \varepsilon_{jt} + \log u_t$

Letting $\log v_{jt}$ be denoted by v'_{jt}, and recalling the above discussion of the two error components ε_{jt} and u_t, the statistical specifications for v'_{jt} are:

(2.6) $$\begin{aligned} &\mathrm{Cov}(v'_{it}, v'_{jt}) = \sigma_{ij} \\ &\mathrm{Cov}(v'_{it}, v'_{jm}) = 0 \quad , \quad t \neq m \end{aligned}$$

We actually make a somewhat stronger assumption than (2.6) based on the following argument. Since we are dealing with data from a randomized experimental design, the following relationship holds in the correlations:

(2.7) $\rho_{ij} = \dfrac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}} = \rho$ for all $i \neq j$ pairs.

In addition, we assume that the standard deviation of y_{jt} is proportional to the mean of y_{jt}, which implies that

(2.8) $\sigma_{ii} = \sigma^2$, $i = 1, 2, \ldots 8$

Assumptions (2.7) and (2.8) together imply that there is only one extra unknown parameter in this model compared to the error specification in the classical regression model, i.e., the correlation parameter for treatments in a given year, designated as ρ in (2.7) above.

There is a complication in estimating the parameters in (2.4) since the parameters enter nonlinearly in a least squares criterion. This problem is handled by a generalized nonlinear least squares algorithm developed by one of the authors for the XDS Sigma 7 computer at M.S.U. The associated statistical theory for this type of model can be found in [Malinvaud, Chapter 9].

The Winter Wheat Equation

In fitting a polynomial yield surface, the shape of the surface frequently does not conform to a priori assumptions. For example, there can easily be local areas where the surface is not concave downward, especially areas considerable distance from the mean point of the independent variable. This problem was encountered in statistically fitting the winter wheat equation.

In particular, the surface was not concave with respect to variation on nitrogen when total water was below about 7.5 inches. This condition gave nonsense results for an optimal nitrogen fertilization decision rule at very low levels of soil water at seeding time. This deficiency was overcome by imposing a linear constraint on the parameter estimates.

The second derivative of (2.1) with respect to N is

$$(2.9)\quad \frac{\partial^2 y}{\partial N^2} = 2(b_2 + 3b_3N + b_8W).$$

The following constraint was imposed,

$$(2.10)\quad b_2 + 5b_8 = 0$$

Referring to (2.9), it is seen that when N = 0, the constraint forces $\partial^2 y/\partial N^2 = 0$ when W = 5. This constraint worked well when the N^3-term was deleted from the equation ($b_3 = 0$). Since the estimate of b_3 was only marginally significant, and the positive estimate of b_3 seemed to merely "flatten out" the response with respect to nitrogen at high levels of nitrogen, this term was deleted in the final model. Considerable experimentation with b_3 included suggested that an additional constraint besides (2.10) would be required if b_3 were left in the equation; thus $b_3 = 0$ is a simple choice for that second constraint.

The final estimate of the yield response equation is given in Table 1. The coefficient of multiple determination is not very meaningful in this situation because of the correlation across treatments; however its value is 0.86. The correlation across treatments is 0.66 which is obviously not trivial.

It is easily shown that when the two error components in (2.5) are uncorrelated (a plausible assumption in this application), the correlation ρ is the ratio of the year to year variance to the total variance, which is 0.66 in our results here. Therefore, about two-thirds of the unexplained variance in the logarithm of yields is due to the random shocks associated with different years. This relationship holds approximately in the natural units of bushels per acre, also.

The parameter estimate for extra nitrogen mineralized during the summer fallow year is 21 pounds, which might seem a little low based on work with chemical tests. However, the statistical precision leaves something to be desired, note the standard error of 8.3 pounds. The "true parameter" could easily be 30 or 35 pounds and be consistent with the statistical results. This particular parameter estimate seemed a little sensitive to the model specification too. One reassuring result is that the barley yield equation gave essentially the same estimate for this parameter.

Significance levels implied by the t-ratios are only approximate for two reasons: (1) the parameters enter the least squares criterion in a nonlinear way; and, (2) the correlation parameter which enters in the generalized least squares criterion is estimated simultaneously with the parameters. A few analyses on what appeared to be marginally significant parameters at the 5 percent levels, suggested that the parameter estimates are typically more significant than indicated by the t-ratios.

A likelihood ratio test can be performed by doing an analysis with and without a parameter (or set of parameters) included. This test showed the fallow effect to be marginally significant at the 0.02 level, and the NW^2 term was marginally significant at the 0.03 level. Both of these levels are smaller than suggested by the t-ratios reported in Table 1.

Table 1

Statistical Results for Winter Wheat Response Surface

Polynomial Term	Regression Coefficient	Standard Error	t-ratio
constant	-2.35	5.22	-0.451
N	-0.417	0.0713	-5.84
N^2	0.00168	0.000397	4.24
W	3.72	0.992	3.75
W^2	-0.140	0.0427	-3.28
NW	0.0838	0.0145	5.77
N^2W	-0.000336	0.0000794	-4.24
NW^2	-0.00110	0.000563	-1.95
Fallow Effect	6.61	2.97	2.23
Mineralized Nitrogen on Fallow:	21.1	8.27	2.56

Correlation among treatments = 0.658
Standard error of estimate = 0.209
(unit is natural logarithm of yield[a/])

46 degrees of freedom

[a/] A comparable measure in the original unit of measure, at mean yield in the sample, is approximately 8.0 bushels per acre. (This is one-half of a 68% confidence interval in the original units.)

The fallow effect is a net effect in addition to benefits from water storage and extra nitrogen mineralized in the soil. Agronomists have conjectured that this net effect is largely a result of improved stand when planting on fallow as compared to seeding in stubble.

A tabular presentation of the fitted yield response surface is given in the Appendix. The interested reader can discern the general structure of the surface from this table. A profit function is also given for wheat price at $3 per bushel and nitrogen price at $0.20 per pound.

It is easy to derive an optimal decision rule for nitrogen fertilization of wheat. By an optimal decision rule, we mean a schedule showing the amount of nitrogen fertilizer to apply for an observed level of soil water at seeding time such that conditional expected profit is maximized. An equation with N the dependent variable and W the independent variable is obtained by solving for N in the following equation,

$$(2.11)\quad P_W \frac{\partial y}{\partial N} = P_N \quad ,$$

where P_W and P_N are prices of wheat and nitrogen; respectively, and $\partial y/\partial N$ is the slope of the yield response surface with respect to nitrogen while holding water fixed. Solution of (2.11) using the notation of (2.1) gives

$$(2.12)\quad N = (P_N/P_W - b_1 - b_6W - b_8W^2)/2(b_2 + b_7W).$$

The next step is to make a change of variable from W, which is total water, to say S, which is soil water. This is done by simply defining W as the sum of soil water and mean precipitation for May through July. Therefore, we use the substitution

(2.13) $W = S + 6.6$

in (2.12) since the long-term mean is 6.6 inches for May-July precipitation in the area of the experiment.

As can be seen from (2.12), only the price ratio, P_N/P_W, is important in determining optimal nitrogen fertilization. The decision rule is given in Table 2 for \$3 wheat and \$0.20 nitrogen ($P_N/P_W = .0667$). It is readily seen from the table that the decision rule is relatively constant for soil water in excess of 3 inches. The results do not change much with wheat prices except at soil water levels below 3 inches.

An examination of the profit function in Appendix Table 1 will show that profit is quite "flat" with respect to variations on nitrogen in the vicinity of maximum profits. Therefore, an error of less than 10 pounds from the optimum rate of nitrogen application will have a trivial cost in lost profits. One might conclude that using Table 2 without regard to prices would be a reasonable approximation to optimal fertilization. Since profit variability tends to increase with nitrogen fertilization, it would be better to error on the low side with respect to applied nitrogen.

The Barley Yield Equation

The barley equation showed declining yields with respect to total water at very low levels of water under the initial estimation. A constraint was introduced which forced incremental yield with respect to water to be zero at the lower bound of the experimental data. The barley response surface was free from any apparent anomalies when this constraint was applied.

Table 2

Nitrogen Fertilizer Decision Rule for Winter Wheat[a]

Soil Water (inches plant available)	Nitrogen Fertilizer (lbs/acre)	
	Stubble	Fallow
1.0	52	32
1.5	59	39
2.0	64	44
2.5	68	48
3.0	71	51
3.5	73	53
4.0	75	55
4.5	76	56
5.0	77	57
5.5	77	57
6.0	78	58
6.5	78	58
7.0	78	58
7.5	78	58
8.0	78	58
8.5	78	58
9.0	78	58
9.5	78	58
10.0	78	58
10.5	77	57
11.0	77	57
11.5	76	56
12.0	76	56
12.5	76	56
13.0	75	55

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[a] Prices are $3 wheat and $0.20 nitrogen

The empirical equation and statistical results are given in Table 3. The low t-ratios on the interaction terms NW, N^2W, and NW^2 suggested a model with only NW as an interaction. Results of this equation are given in Table 4. A comparison of results in Tables 3 and 4 would suggest that the two interactions N^2W and NW^2 could be deleted, but a likelihood ratio test indicated that these two terms together are marginally significant at the 10 percent level. The decision was made to use the results of Table 3, with all the interaction terms, in the economic analysis.

The standard error of the estimate is much higher for barley than wheat, indicating a poorer explanation of barley yield variation. This is also suggested by the coefficient of multiple determination which is 0.59 for barley compared to 0.86 for wheat. Nevertheless, the allocation of unexplained variance between treatments and year to year shocks is about equal for the two crops since the correlation across treatments is 0.64 for barley and 0.66 for winter wheat.

The net effect of planting on fallow instead of stubble is considerably greater for barley than winter wheat, 14.7 bushels per acre instead of 6.6. Estimated amount of nitrogen mineralized during the year of fallow is nearly the same as estimated by the winter wheat equation, 19 versus 21 pounds per acre, which is reassuring.

A decision rule for nitrogen fertilization of barley is given in Table 5 for a barley price of $1.90 per bushel and nitrogen price of $0.20 per pound. As in the case of winter wheat fertilization, the profit function is quite "flat" with respect to variations in nitrogen, holding water constant. Therefore, little will be lost by using the rule in Table 5

Table 3

Statistical Results for Barley Yield Response Surface

Polynomial Term	Regression Coefficient	Standard Error	t-ratio
constant	99.0	19.5	5.07
N	0.208	0.407	0.510
N^2	0.00106	0.00278	0.381
W	- 24.8	6.43	-3.87
W^2	2.47	0.634	3.90
W^3	- 0.0724	0.0183	-3.95
NW	- 0.0317	0.0577	-0.549
N^2W	- 0.000314	0.000238	-1.32
NW^2	0.00299	0.00209	1.43
Fallow Effect	14.7	3.45	4.26
Mineralized Nitrogen on Fallow:	18.8	10.9	1.72

Correlation among treatments = 0.642
Standard error of estimate = 0.304
(unit is natural logarithm of yield[a])

46 degrees of freedom

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[a] A comparable measure in the original unit of measure, at mean yield in the sample, is 13.1 bushels per acre.

Table 4

Alternative Model for Barley Yield Response Surface

Polynomial Term	Regression Coefficient	Standard Error	t-ratio
constant	81.9	13.7	5.96
N	0.120	0.116	1.03
N^2	- 0.00247	0.000993	-2.49
W	-19.6	4.50	-4.35
W^2	1.93	0.437	4.43
W^3	- 0.0558	0.0122	-4.56
NW	0.0137	0.00650	2.11
Fallow Effect	14.5	3.29	4.41
Mineralized Nitrogen on Fallow:	18.5	13.4	1.38

Correlation among treatments = 0.645
Standard error of estimate = 0.308

48 degrees of freedom

Table 5

Nitrogen Fertilizer Decision Rule for Barley[a]

Soil Water (inches plant available)	Nitrogen Fertilizer (lbs/acre)	
	Stubble	Fallow
1.0	13	0
1.5	14	0
2.0	16	0
2.5	17	0
3.0	19	0
3.5	21	1
4.0	23	3
4.5	25	5
5.0	27	7
5.5	29	9
6.0	31	11
6.5	33	13
7.0	35	15
7.5	37	17
8.0	39	19
8.5	42	22
9.0	44	24
9.5	46	26
10.0	48	28
10.5	51	31
11.0	53	33
11.5	55	35

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[a] Prices are $1.90 barley and $0.20 nitrogen

even when prices do not correspond; remember it is the price ratio P_N/P_W which determines the optimal rate of nitrogen. A price of $1.93 was the ten-year average price for barley in constant 1976 dollars for the period 1966-75, and the counterpart for winter wheat was $3.04 per bushel. These are average market prices received by farmers in Montana.

Analysis of Cropping Strategies

First of all, we must devise some method to make the experimental yield equations comensurate with experienced farm yields. Proportional adjustments of the equations would seem to make the most sense. A recent study of production costs for Chouteau County, based on information from a panel of progressive farmers, gave panel estimates of 35 and 40 bushels per acre for winter wheat and barley, respectively. These yields are used to determine a factor for adjustment. Both yields are for crops planted on fallow.

Mean yields for planting on fallow are estimated from the experimental yield equations by averaging predicted yields for each year of the experiment. Predicted yields are calculated by determining the optimal amount of nitrogen fertilizer to apply for observed soil water at seeding time during a given year, and then taking total water as the sum of soil water and mean precipitation for May through July. This procedure provides a longer period than the experiment for averaging across seasonal precipitation levels, but must rely on only 7 years of data for plant available soil water after a year of fallow. Some improvement could be made in the procedure by fitting a frequency curve to soil water measurements, but it was considered rather questionable in view of the extra computation.

Average yields obtained by the above procedure for the fitted equations were 53.6 and 61.9 bushels per acre for winter wheat and fallow, respectively. If we adjust winter wheat yield downward by a factor so that yield equals 35 bushels, the factor is 0.65, i.e., (0.65)(53.6) = 35. Rather surprisingly, a comparable adjustment on barley yield gives the same factor (to two significant digits).

The yield equations applicable for farm yields are as given in Tables 1 and 3 for winter wheat and barley, respectively, except that each equation is multiplied by 0.65. In the economic analysis to follow, we can adjust price of the product downward by the factor 0.65 and use the experimental yield equations directly because gross revenue is adjusted the same either way.

Soil Moisture Analysis

We need an estimate of plant available soil water frequencies at seeding time in the fall. The only data are for seven years during the experiment; the mean for that period was 4.23 inches and the standard deviation was 2.59. A symmetric distribution like the normal is inappropriate because of truncation at zero, less than two standard deviations from the mean. We also have truncation at field capacity, but to five feet for this soil; the upper bound is about 13 inches which is considerably farther from the mean than the lower bound.

The small number of observations hardly warranted an elaborate evaluation of various functional forms; the power of any test for goodness of fit would be extremely low. Essentially a log-normal distribution was used with truncation at the upper bound of soil water capacity. A small change from the truncated log-normal distribution was made to better deal with the small sample available.

After a logarithmic transformation (natural base), mean plant available water was 1.22 and the standard deviation was 0.795. Let x be any plant available soil water measurement drawn at random, and suppose the mean of x is μ_x and s_x is the usual sample estimate of the standard deviation of x. Then, if x is normally distributed,

$$\frac{x - \mu_x}{s_x} = t$$

is distributed according to the t-distribution with degrees of freedom one less than sample size. For x defined as the natural logarithm of plant available soil water, we use

$$\frac{x - 1.22}{0.795} = t \ , \qquad (2.14)$$

and assume that the t-distribution with six degrees of freedom is a good approximation. If x were normal, the sample mean $\bar{x}$ has been used in place of the actual mean, μ_x, as an approximation. We used (2.14) instead of the standardized normal because the small number of degrees of freedom gives a very imprecise estimate of the standard deviation, and it was thought the t-distribution fitted as described would be better. A tabular approximation to the fitted distribution is given in Table 6.

A comparable procedure is required to estimate the probability distribution for plant available soil water in the spring after a crop of winter wheat. The mean of soil water was 5.8 inches, and the standard deviation was 2.19, for the spring data at seeding time (same seven years). Notice

Table 6

Probability Distribution for Autumn Soil Water After a Crop of Barley

Water Interval (inches)	Midpoint (inches)	Probability	Cumulative Probability (upper bound of interval)
0-0.5	0.25	.0284	.0284
0.5-1.0	0.75	.0660	.0944
1.0-1.5	1.25	.0914	.1858
1.5-2.0	1.75	.1005	.2863
2.0-2.5	2.25	.0988	.3851
2.5-3.0	2.75	.0906	.4757
3.0-3.5	3.25	.0795	.5552
3.5-4.0	3.75	.0681	.6233
4.0-4.5	4.25	.0584	.6817
4.5-5.0	4.75	.0490	.7307
5.0-5.5	5.25	.0419	.7726
5.5-6.0	5.75	.0352	.8078
6.0-6.5	6.25	.0298	.8376
6.5-7.0	6.75	.0253	.8629
7.0-7.5	7.25	.0217	.8846
7.5-8.0	7.75	.0187	.9033
8.0-8.5	8.25	.0173	.9206
8.5-9.0	8.75	.0129	.9335
9.0-9.5	9.25	.0123	.9458
9.5-10.0	9.75	.0109	.9567
10.0-11.0	10.5	.0178	.9745
11.0-12.0	11.5	.0142	.9887
12.0-13.0	12.5	.0113	1.0000

that the mean is larger and the standard deviation smaller than for the fall measurements. Measurements in the spring are only to four feet instead of five feet as in the fall. Total capacity of the soil to four feet is about 1. inches. Notice that the sample mean of 5.8 is about half way between the two bounds.

In view of the apparent symmetry for soil water measurements in the spring, we did not transform the data to logarithms, but applied a truncated t-distribution to the raw data. Truncation was at both ends of the data, and arbitrarily chosen symmetrically around the mean with a total range of 10 inches. Thus the soil water variable is confined to the interval 5.8 ± 5 inches. A tabular approximation to the distribution is given in Table 7.

Basic Trade-Off Equation

A formal mathematical optimization model can be used to evaluate various cropping strategies (see Burt and Allison, or Burt and Johnson), but we use an approximation here, which in this particular application, turns out to give the same results as more elaborate methods would have. One reason for the simplification is that soil water is essentially independent from year to year at the experimental site; that is to say, the soil water measurement this fall has no influence on the same measurement next fall. A similar result holds for spring soil water measurements. Simple correlations on water measurements in adjacent years for each spring and fall gave negative correlations which were essentially zero, giving no support for a positive correlation as expected. There was a positive correlation between spring soil water and fall soil water of

Table 7

Probability Distribution for Spring Soil Water
After a Crop of Winter Wheat

Water Interval (inches)	Midpoint (inches)	Probability	Cumulative Probability (upper bound of interval)
0.8-1.3	1.05	.0123	.0123
1.3-1.8	1.55	.0171	.0294
1.8-2.3	2.05	.0233	.0527
2.3-2.8	2.55	.0313	.0840
2.8-3.3	3.05	.0412	.1252
3.3-3.8	3.55	.0530	.1782
3.8-4.3	4.05	.0654	.2436
4.3-4.8	4.55	.0772	.3208
4.8-5.3	5.05	.0871	.4079
5.3-5.8	5.55	.0921	.5000
5.8-6.3	6.05	.0921	.5921
6.3-6.8	6.55	.0871	.6792
6.8-7.3	7.05	.0772	.7564
7.3-7.8	7.55	.0654	.8218
7.8-8.3	8.05	.0530	.8748
8.3-8.8	8.55	.0412	.9160
8.8-9.3	9.05	.0313	.9473
9.3-9.8	9.55	.0233	.9706
9.8-10.3	10.05	.0171	.9877
10.3-10.8	10.55	.0123	1.000

0.58, but our analysis is unaffected by this relationship.

One side of the trade-off equation is average annual net returns from an acre of land when an optimal strategy is followed. As an approximation for this value, we use average annual net returns from the alternate winter wheat-fallow rotation. If it should turn out that this rotation is superior to any cropping strategy based on soil water at seeding time, then our approximation is perfect.

Fixed costs do not enter into the decision in making year to year evluations of which crop to plant, so we deduct only variable costs to get net returns. The following notation is introduced:

m = plant available soil water (inches)

P_w = price of winter wheat per bushel (dollars)

P_n = price of fertilizer nitrogen per pound (dollars)

$c(m)$ = a function describing the cost of fertilizer nitrogen when planting on stubble (dollars per acre)

z = variable costs of production, except nitrogen fertilizer costs are excluded (dollars per acre)

C_f = variable costs of a year in summer fallow (dollars per acre)

$y(m)$ = conditional expected yield of winter wheat planted on stubble when plant available soil water is m inches at seeding (bushels per acre)

$\bar{y}_f$ = average yield of winter wheat when planted on fallow (bushels per acre)

$\bar{c}_f$ = average cost of nitrogen fertilizer when planting on fallow (dollars per acre)

Nitrogen costs, $c(m)$ and $\bar{c}_f$, are functions of the price ratio P_n/P_w since optimum nitrogen fertilization is assumed, but for simplicity in the notation, the price ratio was not included in the functional forms. Also, the price ratio is implicit in yields $y(m)$ and $\bar{y}_f$.

Elementary reasoning would tell us that when soil water is high enough in the fall that expected net returns from planting a crop of wheat exceeds average annual net returns from the best strategy, then we should plant the wheat and vice versa. The condition where we would be indifferent is when the expected net returns from planting are just equal to average net returns under the best strategy. We are using alternate winter wheat-fallow as an approximation for the best strategy.

Therefore, our trade-off equation is

$$(2.15)\quad P_w y(m) - c(m) - z = [P_w \bar{y}_f - z - C_f - \bar{c}_f]/2$$

The left hand side of (2.15) is expected net returns from planting a crop of winter wheat when observed soil water is m. The right hand side is average annual net returns in alternate winter wheat-fallow; division by two allows for a crop every other year.

We transpose z on the left to the right side and simplify to get

$$(2.16)\quad P_w y(m) - c(m) = z/2 + [P_w \bar{y}_f - C_f - \bar{c}_f]/2$$

The above equation, with m as the unknown variable, is solved to determine the threshold level of soil water above which winter wheat is planted on stubble, and below which, a year of summer fallow is decided upon. This result is specific to given prices, P_w and P_n.

The above discussion would lead one to assume that we are considering planting winter wheat on winter wheat stubble, but this is not the case. The previous crop would have to have been barley or some other crop like safflower that left a light enough stubble to make planting winter wheat feasible in this climate.

The analysis was done with the following estimates of costs and price of nitrogen:

$z = \$20$

$C_f = \$5$

$P_n = \$0.20$

The remaining terms in (2.16) depend on the winter wheat price specified. Variable costs of production were taken from (Schaefer, et al. 1976c) adjusted by deleting the following items from variable costs: (1) nitrogen fertilizer; (2) crop insurance; and, (3) labor. Variable costs of fallow were taken from the same source with labor costs deducted.

The analysis for a spring decision on whether to plant barley after a year with the land in crop is essentially the same. The only difference is in the interpretation of the left hand side of (2.16); $y(m)$ is then barley yield and P_w is replaced with barley price.* It is assumed that costs of production, z, are the same for barley as wheat. Also, the nitrogen fertilizer cost function, $c(m)$, must be reinterpreted as that appropriate for optimum barley fertilization.

"Optimum" nitrogen fertilization was determined with the scaled down yield equation which makes the rates lower than given in Tables 2 and 5 for winter wheat and barley, respectively. Also, the analysis was done for several prices of wheat and barley.

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*See page 38-a.

*After the following analysis was completed, it was realized that the conditional expected net returns, given the land is in barley stubble in the fall, is greater than average annual returns from alternate wheat-fallow by a significant amount (about $5 per acre with wheat at $3 per bushel). In view of this fact, a more precise trade-off equation would be to change the left hand side of (2.15) to the average of net returns from barley (given soil moisture) and expected net returns from the following year if the land in the autumn were in barley stubble. This two-year average would introduce the extra advantage which planting barley has on expected net returns as viewed in the fall after the barley has been harvested. The difference in results is quite trivial as far as estimating an optimal decision rule based on an expected profit criterion, but the measure of expected opportunity loss used below to estimate costs of deviating from the "otpimal" strategy is significantly affected. This difference is quantified at the end of the report.

Relative Price of Barley to Wheat

Economic strategies for cropping decisions obviously depend critically on relative prices of barley and wheat. A regression was run with barley price the dependent variable and winter wheat price the independent variable (an arbitrary choice). In the raw data for ten years (1966-75), using average prices received by Montana producers, the correlation was 0.978 which suggests a strong systematic relationship between the two prices. However, general changes in the price level tend to exaggerate the correlation in prices.

Consequently, the prices were deflated by prices paid by farmers for production items, and the correlation was 0.958, still quite substantial. Measured in 1976 dollars, the standard error of the estimate for barley price, given winter wheat price, was $0.18. The fitted regression equation was

(2.17) $P_b = 0.455 + 0.484P_w$,

where P_b and P_w are barley and winter wheat prices, respectively. Mean prices (1976 dollars) were $3.04 and $1.93 for winter wheat and barley, respectively. Selected pairs of prices from (2.17) are given below.

Table 8

Relative Prices of Wheat and Barley

P_w	2.50	3.00	3.50	4.00	4.50	5.00
P_b	1.67	1.91	2.15	2.39	2.63	2.88

Relative prices may deviate considerably from the relationship of (2.17), but for planning purposes and economic analysis such as being pursued here, prices in the above table should be a good guide. The analysis is performed for the above prices from \$2.50 to \$4.00 for winter wheat.

Criterion of Maximum Average Profits

The trade-off equation (2.16) was solved for each of the four wheat prices and the results are summarized in Table 9. These results constitute the "optimal" decision rule for decisions in the fall when the land is in barley stubble so that planting winter wheat is feasible. By "optimal," we mean within the context of the model and the empricial yield response functions which are being used for the analysis. In actual farm circumstances, yields from alternate winter wheat-fallow might be lower than if barley were worked into the rotation. These complications are introduced later.

For the spring decision and a criterion of maximum average profits, solution of (2.16), adjusted on the left hand side for barley, gives a critical soil moisture level of 8.9 inches when wheat price is \$2.50 and barley price is \$1.67. At the other pairs of prices in Table 8 (up to \$4 for wheat price), barley would never be planted. That is to say, barley would not be superior to alternate winter wheat-fallow at any observed soil water level in the spring.

Table 9

Autumn Decision Rule Maximizing Mean Profits

Wheat Price	Critical Soil Water Level[a]	Right Hand Side of Equation
(dollars)	(inches)	(dollars)
2.50	4.0	46.00
3.00	3.5	54.50
3.50	3.2	63.25
4.00	3.0	71.75

[a] The decision rule is plant winter wheat if plant available soil water exceeds the critical level and summer fallow if it is below the critical level.

Expected Opportunity Loss Analysis

Winter wheat yields can be expected to increase when barley is introduced into the cropping sequence because of benefits from disease and weed control, as well as other more diverse agronomic benefits. This is particularly true under actual farm conditions as contrasted to the more intensive controls imposed under experimental conditions. Consequently, the rather simple comparisons made with the economic trade-off equation of (2.16) could easily result in barley not being planted frequently enough.

The following analysis is directed at estimating the average opportunity loss associated with various decision rules, where opportunity loss is the average difference in net returns under the "optimal" decision rule according to (2.16) and a specific decision rule which plants barley with a greater frequency and also plants winter wheat on barley stubble with a greater frequency. The average opportunity loss can then be evaluated in the light of prospective winter wheat yield increases and control of saline seep problems.

A decision rule is specified by two numbers: (1) the critical value of autumn soil water and (2) the critical value of spring soil water. Let us call these two critical values M_a and M_s for the autumn and spring decisions, respectively. We use the convention $M_s = \infty$ to mean that barley is never planted in the spring. Recall that a crop is planted if plant available soil water exceeds the critical value and vice versa.

For any pair of critical values (M_a, M_s), we can use the probability Tables 6 and 7 to obtain the probabilities of planting crops in the autumn and spring. For example, take $M_a = 2.0$ and $M_s = 5.3$; from Table 6, it is seen that the probability of not planting winter wheat on stubble is 0.2863 (last column, line 4); likewise, the probability of not planting barley on stubble in the spring is 0.4079 (last column of Table 7, line 9). The probabilities of planting crops are obtained by subtracting these probabilities from one.

The restriction is made in the analysis here that winter wheat will always be planted after a year of summer fallow. This specification is warranted by a simple comparison of average net returns from each barley and winter wheat planted on fallow. An examination of the paired prices of winter wheat and barley in Table 8 suggests that the higher prices, the more advantage winter wheat will have. A comparison of mean net returns at prices corresponding to average prices (1976 dollars) over the period 1966-75 shows a difference of about $25 per acre in favor of winter wheat. This result is with yields scaled down from experimental yields to 35 and 40 bushels per acre for winter wheat and barley, respectively. Even at $2.50 wheat and $1.67 barley prices, the difference is about $17 per acre. Of course, something like malting barley prices could change this substantially for any given year and producer. Nevertheless, this evidence is rather convincing as a general rule.

The cropping systems strategy can be summarized as follows:

1. Plant winter wheat on summer fallowed land;
2. Plant winter wheat on barley stubble if plant available soil water exceeds M_a; and,
3. Plant barley on winter wheat stubble if plant available soil water exceeds M_s.

The state of land use in any particular year is either fallow, winter wheat, or barley.

Continuing with the example where $M_a = 2.0$ and $M_s = 5.3$, a set of conditional probabilities can be defined for land use next year for a given land use this year. These probabilities are given for this example in Table 10. The first row is obvious since winter wheat is always planted on fallow. The probabilities in the second row emanate from the probability of soil water exceeding or not the value $M_s = 5.3$. As indicated earlier 0.4079 is the probability of soil water not exceeding 5.3 in the spring (read from Table 7), and is thus the probability of a year of fallow after a year of winter wheat. Its complement, 0.5921, is the probability of planting barley after winter wheat, and there is no chance of a second year of winter wheat. The probability of a year of fallow after barley is the probability of soil water in the fall not exceeding $M_a = 2.0$ (read from Table 6) which was obtained earlier as 0.2863. Its complement is the probability of planting winter wheat after a year of barley, and again, the chance of a second year of the same crop is zero.

Table 10

Conditional Probabilities for Land Use
With $M_a = 2.0$ and $M_s = 5.3$

Current Land Use	Probabilities of Land Use Next Year		
	Fallow	Wheat	Barley
Fallow	0	1.0	0
Wheat	0.4079	0	0.5921
Barley	0.2863	0.7137	0

When land use from year to year obeys a probabilistic pattern described by a table of conditional probabilities, like the example problem in Table 10, it is called a Markov chain. By using the elementary principles of probability theory, the probability of land use at any future date can be calculated for any given land use at the present time. An important property of such Markov chains is the existence of a set of equilibrium probabilities which have the following interpretation. If a year far into the future were picked at random, probabilities of land use would be independent of current land use, and these probabilities are unique as well as easily calculated from a table such as Table 10 (Kemeny, Snell, and Thompson).

These equilibrium probabilities for a specified decision rule will be referred to as relative frequencies of land use because they measure the long-run relative frequencies of land use, regardless of current land use. These relative frequencies are the statistical counterpart of the proportions of time that land is in various uses under a conventional rotation. If you please, think of relative frequencies of land use as describing a probabilistic rotation. One word of caution, however, the same relative frequencies could be associated with more than one cropping strategy (decision rule) and the economic consequences of these strategies would not be the same. In other words, relative frequencies of land use do not reveal all of the information and consequences of a particular strategy.

In the analysis of Markov chains for this research, the relative frequencies were calculated by taking powers of the matrices of probabilities (Kemeny, et al). The matrices are only three dimensional and n iterations give a matrix to the power of 2^n, which makes the method fast and simple for computer use. Fewer than ten iterations were required to get convergence which is accomplished when all the rows of the computed matrix are identical.

Relative frequencies for the example given in Table 10 are 27 percent fallow, 46 percent winter wheat, and 27 percent barley. Approximately the same frequencies would prevail under a fixed four year rotation of fallow, winter wheat, barley, winter wheat, but the implications for soil water and saline seep control would be much different. The decision rule associated with Table 10 implies that fallow will be used only during the driest years as reflected by soil water in the fall and spring, but the fixed four-year rotation completely ignores soil water at seeding time as a criterion for fallowing. Average deep water percolation through the root zone would be higher under the fixed rotation than under the decision rule which uses soil water as a criterion for fallow.

The decision rules for the spring decision were $M_s = \infty$, except at a price of \$2.50 for wheat and \$1.67 for barley. Even then, $M_s = 8.9$ which implies fallow after winter wheat about 92 percent of the time (see Table 7 last column and fifth row up from the bottom). Recall $M_s = \infty$ implies to always fallow after winter wheat. These results essentially lock the producer into winter wheat-fallow with barley planted only on rare occasions when prices are low and soil water is very high in the spring.

Conditional expected opportunity losses can be calculated for violating the maximum expected profit decision rule. We are only interested in violations which mean planting a crop with "too little" soil water. For example, let plant available soil water be 3.75 inches in the fall after a crop of barley, and suppose wheat price is $2.50. From Table 9, it is seen that the "optimum" value of M_a is 4.0, so an observed level of water equal to 3.75 would imply fallowing if we followed the decision rule. What would be the average opportunity cost of planting winter wheat instead of fallowing?

Returning to our trade-off equation (2.16), setting m = 3.75 will result in the left hand side being smaller than the right hand side; this difference is conditional expected opportunity loss. The computed difference is $1.44. Turning to Table 7, note that 3.75 is the midpoint of the soil water interval 3.5 to 4.0 inches which occurs with probability 0.0681. Therefore, reduction of M_a from 4.0 to 3.5 would result in an expected opportunity loss of

$$(0.0681)(1.44) = \$0.10.$$

Observe the distinction between conditional and unconditional expected opportunity loss; we dropped the redundant adjective unconditional though.

By using the procedure described above for computing conditional expected opportunity loss and expected opportunity loss, Table 11 was constructed which provides a running account of expected losses as M_a is systematically reduced from the values of Table 9. Notice how conditional expected opportunity losses increase more rapidly than cumulative expected losses. In this context, conditional expected loss is like an incremental

Table 11

Expected Opportunity Losses for Autumn Decision Rules

Soil Water Interval	Midpoint	Probability	$2.50 Wheat[a/] Conditional Expected Loss	$2.50 Wheat[a/] Cumulative Expected Loss	$3.00 Wheat[b/] Conditional Expected Loss	$3.00 Wheat[b/] Cumulative Expected Loss	Probability of Planting Wheat[c/]
(inches)	(inches)		(dollars)	(dollars)	(dollars)	(dollars)	
3.5 to 4.0	3.75	.0681	1.44	.10	0	0	.4448
3.0 to 3.5	3.25	.0795	4.14	.43	2.14	.17	.5243
2.5 to 3.0	2.75	.0906	6.92	1.05	5.63	.68	.6149
2.0 to 2.5	2.25	.0988	9.74	2.02	9.20	1.59	.7137

[a/] Critical soil water level is 4 inches and right hand side of trade-off equation is $46.

[b/] Critical soil water level is 3.5 inches and right hand side of trade-off equation is $54.50.

[c/] Probability corresponds to using a critical soil water level equal to the bottom of the interval.

Table 11 (cont'd)

Expected Opportunity Losses for Autumn Decision Rules

Soil Water Interval	Midpoint	Probability	\$3.50 Wheat[d] Conditional Expected Loss	\$3.50 Wheat[d] Cumulative Expected Loss	\$4.00 Wheat[e] Conditional Expected Loss	\$4.00 Wheat[e] Cumulative Expected Loss	Probability of Planting Wheat[c]
(inches)	(inches)		(dollars)	(dollars)	(dollars)	(dollars)	
3.0 to 3.2	3.35	.0318	.95	.03	0	0	.5243
2.5 to 3.0	2.75	.0906	4.07	.39	2.46	.22	.6149
2.0 to 2.5	2.25	.0988	8.41	1.23	7.52	.97	.7137

[c] Probability corresponds to using a critical soil water level equal to the bottom of the interval.

[d] Critical soil water level is 3.2 inches and right hand side of trade-off equation is \$63.25.

[e] Critical soil water level is 3.0 inches and right hand side of trade-off equation is \$71.75.

loss in going to a drier and drier decision rule, while cumulative expected losses are weighted average losses over the interval of soil water where wheat is being planted in violation of the decision rule in Table 9. If M_a is chosen as the lower bound of any interval in the first column of Table 11, cumulative expected loss for that row is expected opportunity loss associated with the autumn decision rule, and the last column of the table is the probability of planting wheat on stubble.

As an example, let us examine Table 11 under a price of $3.00 for wheat. In the soil water interval 3.5 to 4.0 inches, there are no conditional expected opportunity losses because the critical soil water level under an average maximum profit decision rule is 3.5 inches which lies at the lower boundary of the interval; losses do not occur unless a decision rule is used where wheat is planted with less than 3.5 inches of plant available water. In the interval 3.0 to 3.5 inches, conditional expected losses are seen to be $2.14 which is the difference between $54.50 and conditional expected net returns from planting barley with plant available water equal to 3.25 inches (midpoint of interval). The $54.50 used as a basis to calculate the loss is the right hand side of the trade-off equation (2.16), which is average net returns in alternate winter wheat-fallow. The last two rows for the conditional expected losses column under $3 wheat is calculated by the same procedure.

Notice how rapidly these conditional expected losses increase as the decision rule is adjusted to lower and lower critical soil water levels, reaching $9.20 for the interval 2.0 to 2.5 inches. This last loss estimate means that if winter wheat were planted on stubble when there were only

2.25 inches (midpoint) of plant available water, the average sacrifice in net income per acre would be \$9.20. Of course, this estimate imputes nothing to improved yields from an integrated strategy that gets barley into the rotation (the stubble has to be barley), nor is any credit being given to improved saline seep control; these aspects are dealt with shortly.

The cumulative expected loss column in Table 11 is simply a running total of the sum of the cross products of the probability column and conditional expected loss column. This cumulative expected loss column can best be interpreted by an example. Again using \$3 wheat, consider the row for interval 2.5 to 3.0 inches of soil water where cumulative expected loss is \$0.68 in Table 11. If the critical soil water level M_a were taken as 2.5 inches, then any year in which barley stubble was available for recropping would have an average net return of \$0.68 per acre below what it would have if the "optimum" critical value of 3.5 inches were used. If the critical soil water level were pushed on down to 2.0 inches, the average sacrifice in such a year after barley would be \$1.59 per acre. These expected opportunity losses in the cumulative expected loss column of Table 11 are conditional on the land being in a stubble such as barley which makes recropping with winter wheat feasible. This "conditional" aspect must not be confused with the conditional expected loss columns of Table 11 which imply a particular soil water interval as well as the existence of barley or other feasible stubble.

The same kind of economic analysis is applicable to the spring decision when a decision must be made on whether to plant barley after winter wheat or to summer fallow the land. The counterpart of Table 11 is Table 12 for the spring decision. Substantially more rows are required because the critical soil water level under an "optimum" decision rule is relatively high; in fact, it is field capacity for all prices except \$2.50 wheat where it is 8.9 inches.

By using the Markov chain framework discussed earlier and Tables 11 and 12, we can evaluate various strategies for cropping decisions. For each pair of critical soil water levels, M_a in the fall and M_s in the spring, a strategy is defined. A set of long-run relative frequencies for land use exists for each strategy, and expected opportunity losses are available in Tables 11 and 12 for land in barley stubble and winter wheat stubble, respectively. Results are summarized in Table 13.

Optimal combinations of M_a and M_s exist in theory in the sense that for a given frequency of fallow, expected opportunity losses are a minimum. These optimal combinations of M_a and M_s would correspond to equal values of conditional expected losses in Tables 11 and 12 at given prices of grain. For example at \$3 wheat, $M_a = 2.0$, and $M_s = 6.3$; this condition is approximately met since conditional expected opportunity losses are \$9.20 and \$9.02 in Tables 11 and 12, respectively. From Table 13, it is seen that this strategy gives a fallow frequency of 33.5 percent and expected loss of \$1.24. We can conclude that for this frequency of fallow and \$3 wheat, no other strategy gives a smaller expected opportunity loss.

However, serious limitations are encountered in applying this principle. First, the results are quite sensitive to prices of grain, and

Table 12

Expected Opportunity Losses for Spring Decision Rules

Soil Water Interval	Midpoint	Probability	$2.50 Wheat and $1.67 Barley[a] Conditional Expected Loss	$2.50 Wheat and $1.67 Barley[a] Cumulative Expected Loss	$3.00 Wheat and $1.90 Barley[b] Conditional Expected Loss	$3.00 Wheat and $1.90 Barley[b] Cumulative Expected Loss	Probability of Planting Barley[c]
(inches)	(inches)		(dollars)	(dollars)	(dollars)	(dollars)	
10.3 to 10.8	10.55	.0123	0	0	1.10	.01	.0123
9.8 to 10.3	10.05	.0171	0	0	.54	.02	.0294
9.3 to 9.8	9.55	.0233	0	0	.54	.04	.0527
8.8 to 9.3	9.05	.0313	0	0	1.04	.07	.0840
8.3 to 8.8	8.55	.0412	.64	.03	1.99	.15	.1252
7.8 to 8.3	8.05	.0530	1.75	.12	3.32	.33	.1782
7.3 to 7.8	7.55	.0654	3.14	.32	4.98	.65	.2436
6.8 to 7.3	7.05	.0772	4.76	.69	6.90	1.18	.3208
6.3 to 6.8	6.55	.0871	6.56	1.26	9.02	1.97	.4079
5.8 to 6.3	6.05	.0921	8.49	2.05	11.28	3.01	.5000
5.3 to 5.8	5.55	.0921	10.48	3.01	13.63	4.26	.5921
4.8 to 5.3	5.05	.0871	12.48	4.10	15.99	5.66	.6792
4.3 to 4.8	4.55	.0772	14.46	5.21	18.31	7.07	.7564

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[a] Right hand side of trade-off equation is $46 and critical soil water level is 8.9 inches.

[b] Right hand side of trade-off equation is $54.50 and critical soil water level is at the upper bound (field capacity).

[c] Probability corresponds to using a critical soil water level equal to the bottom of the interval.

Table 12 (cont'd)

Expected Opportunity Losses for Spring Decision Rules

Soil Water Interval	Midpoint	Probability	$3.50 Wheat and $2.15 Barley[d/]		$4.00 Wheat and $2.39 Barley[e/]		Probability of Planting Barley[c/]
			Conditional Expected Loss	Cumulative Expected Loss	Conditional Expected Loss	Cumulative Expected Loss	
(inches)	(inches)		(dollars)	(dollars)	(dollars)	(dollars)	
10.3 to 10.8	10.55	.0123	1.79	.02	2.69	.03	.0123
9.8 to 10.3	10.05	.0171	1.22	.04	2.10	.07	.0294
9.3 to 9.8	9.55	.0233	1.28	.07	2.23	.12	.0527
8.8 to 9.3	9.05	.0313	1.91	.13	2.98	.21	.0840
8.3 to 8.8	8.55	.0412	3.05	.26	4.29	.39	.1252
7.8 to 8.3	8.05	.0530	4.62	.50	6.08	.71	.1782
7.3 to 7.8	7.55	.0654	6.55	.93	8.27	1.25	.2436
6.8 to 7.3	7.05	.0772	8.79	1.61	10.80	2.09	.3208
6.3 to 6.8	6.55	.0871	11.25	2.59	13.57	3.27	.4079
5.8 to 6.3	6.05	.0921	13.87	3.87	16.52	4.79	.5000
5.3 to 5.8	5.55	.0921	16.58	5.39	19.58	6.59	.5921
4.8 to 5.3	5.05	.0871	19.31	7.08	22.65	8.57	.6792
4.3 to 4.8	4.55	.0772	22.00	8.77	25.68	10.55	.7564

[c/] Probability corresponds to using a critical soil water level equal to the bottom of the interval.

[d/] Right hand side of trade-off equation is $63.25 and the critical soil water level is field capacity.

[e/] Right hand side of trade-off equation is $71.75 and the critical soil water level is field capacity.

Table 13

Relative Frequencies and Expected Opportunity Losses

Strategies		Relative Frequencies			Expected Opportunity Losses			
M_a	M_s	Fallow	Wheat	Barley	\$2.50 Wheat	\$3.00 Wheat	\$3.50 Wheat	\$4.00 Wheat
(inches)		(percentages)			(dollars)	(dollars)	(dollars)	(dollars)
2.5	7.3	40.6	47.8	11.6	0.28	0.39	0.49	0.62
2.5	6.8	37.8	47.1	15.1	0.48	0.66	0.82	1.02
2.0	6.8	36.9	47.8	15.3	0.64	0.81	0.96	1.15
2.5	6.3	34.7	46.4	18.9	0.78	1.04	1.28	1.56
2.0	6.3	33.5	47.2	19.3	0.99	1.24	1.46	1.73
2.0	5.8	30.0	46.7	23.3	1.43	1.78	2.09	2.46
2.0	5.3	26.6	46.1	27.3	1.94	2.40	2.82	3.30
2.0	5.0	24.7	45.8	29.5	2.26	2.79	3.28	3.83
2.0	4.8	23.5	45.6	30.9	2.49	3.07	3.61	4.21
2.0	4.3	20.8	45.1	34.1	3.04	3.73	4.38	5.09

Table 12 (cont'd)

Expected Opportunity Losses for Spring Decision Rules

Soil Water Interval	Midpoint	Probability	$3.50 Wheat and $2.15 Barley[d/] Conditional Expected Loss	$3.50 Wheat and $2.15 Barley[d/] Cumulative Expected Loss	$4.00 Wheat and $2.39 Barley[e/] Conditional Expected Loss	$4.00 Wheat and $2.39 Barley[e/] Cumulative Expected Loss	Probability of Planting Barley[c/]
(inches)	(inches)		(dollars)	(dollars)	(dollars)	(dollars)	
10.3 to 10.8	10.55	.0123	1.79	.02	2.69	.03	.0123
9.8 to 10.3	10.05	.0171	1.22	.04	2.10	.07	.0294
9.3 to 9.8	9.55	.0233	1.28	.07	2.23	.12	.0527
8.8 to 9.3	9.05	.0313	1.91	.13	2.98	.21	.0840
8.3 to 8.8	8.55	.0412	3.05	.26	4.29	.39	.1252
7.8 to 8.3	8.05	.0530	4.62	.50	6.08	.71	.1782
7.3 to 7.8	7.55	.0654	6.55	.93	8.27	1.25	.2436
6.8 to 7.3	7.05	.0772	8.79	1.61	10.80	2.09	.3208
6.3 to 6.8	6.55	.0871	11.25	2.59	13.57	3.27	.4079
5.8 to 6.3	6.05	.0921	13.87	3.87	16.52	4.79	.5000
5.3 to 5.8	5.55	.0921	16.58	5.39	19.58	6.59	.5921
4.8 to 5.3	5.05	.0871	19.31	7.08	22.65	8.57	.6792
4.3 to 4.8	4.55	.0772	22.00	8.77	25.68	10.55	.7564

[c/] Probability corresponds to using a critical soil water level equal to the bottom of the interval.

[d/] Right hand side of trade-off equation is $63.25 and the critical soil water level is field capacity.

[e/] Right hand side of trade-off equation is $71.75 and the critical soil water level is field capacity.

Table 13

Relative Frequencies and Expected Opportunity Losses

Strategies		Relative Frequencies			Expected Opportunity Losses			
M_a	M_s	Fallow	Wheat	Barley	$2.50 Wheat	$3.00 Wheat	$3.50 Wheat	$4.00 Wheat
(inches)		(percentages)			(dollars)	(dollars)	(dollars)	(dollars)
2.5	7.3	40.6	47.8	11.6	0.28	0.39	0.49	0.62
2.5	6.8	37.8	47.1	15.1	0.48	0.66	0.82	1.02
2.0	6.8	36.9	47.8	15.3	0.64	0.81	0.96	1.15
2.5	6.3	34.7	46.4	18.9	0.78	1.04	1.28	1.56
2.0	6.3	33.5	47.2	19.3	0.99	1.24	1.46	1.73
2.0	5.8	30.0	46.7	23.3	1.43	1.78	2.09	2.46
2.0	5.3	26.6	46.1	27.3	1.94	2.40	2.82	3.30
2.0	5.0	24.7	45.8	29.5	2.26	2.79	3.28	3.83
2.0	4.8	23.5	45.6	30.9	2.49	3.07	3.61	4.21
2.0	4.3	20.8	45.1	34.1	3.04	3.73	4.38	5.09

second, the criterion often implies extremely small or negative values for M_a at low fallow frequencies which are of interest. A practical difficulty is considerable interpolation in Tables 11 and 12. For these reasons, we do not impose this efficiency criterion on the strategies examined, but merely list several strategies in decreasing order according to frequency of fallow (see Table 13). Choosing M_a less than 2.0 inches was considered infeasible on practical grounds because of the hazards of getting a stand of winter wheat. If this constraint is accepted as valid, there is no opportunity for application of the efficiency criterion when frequency of fallow is less than about 33 percent.

Agricultural research indicates considerable advantage in avoiding monoculture because of various disease and weed problems. The practice of alternative winter wheat-fallow has met with considerable success in spite of the hazards of disease and weeds, but there also appears to be considerable opportunity for increased yields by getting a spring planted crop into the cropping system.

Bringing barley into the rotation would help control cheatgrass which has a life cycle corresponding closely with winter wheat. There would also appear to be some advantage for the control of wild oats. An occasional crop of barley would tend to control some of the diseases common to winter wheat, especially soil borne diseases.

Let us suppose for a moment that bringing barley into the cropping system with an average frequency of 20 percent or more would increase average winter wheat yields by two bushel per acre, a rather modest amount. In examining the results of the strategy $M_a = 2.0$ and $M_s = 5.0$ in Table

13, notice that expected opportunity losses across all prices are just a little less than the price of a bushel of wheat. Also, observe that winter wheat is planted with a relative frequency of 46 percent. Consequently, the two bushel increase in yield is experienced on only 46 percent of the cropland, on the average. With \$3 wheat, that increases is equal to \$2.76 per acre annually (2 x 3 x 0.46 = 2.76), and the expected opportunity loss is \$2.79 (see Table 13). Thus it is seen that a two bushel increase in average winter wheat yield attributable to barley in the cropping system can compensate for the expected opportunity loss associated with the strategy $M_a = 2.0$ and $M_s = 5.0$.

Actually, the expected opportunity loss is somewhat lower than the methodology just presented would lead us to believe. There is a rather subtle dynamic element involved in the trade-off between fallowing and planting barley in the spring which is not accounted for by the method used to calculate expected opportunities loss. The additional complexity was discovered as this final report was being typed; time and resource limitations did not permit an entire new analysis. However, some limited calculations for the \$3 wheat case showed expected opportunity losses to be about one dollar for the rule $M_a = 2.0$ and $M_s = 5.0$. For this strategy, it would seem safe to claim that expected opportunity losses are probably covered by a one bushel per acre increase in winter wheat yields which can be attributed to getting barley into the rotation (rather than a two bushel increase as suggested earlier).

The measure of expected opportunity loss used in Table 13 would be appropriate in a somewhat drier location where land in barley stubble in the fall does not have any higher conditional expected net return than average annual returns from alternate wheat-fallow. Consequently, the two-bushel per acre yield increase on winter wheat required to compensate for expected opportunity loss associated with using the strategy $M_a = 2.0$ and $M_s = 5.0$ would be appropriate for somewhat drier winter wheat areas of North Central Montana. As an informed guess, we might say that for areas in this general region which have 14 or more inches of average annual precipitation, a two-bushel increase in winter wheat yields attributable to barley in the cropping sequence would economically justify the strategy $M_a = 2.0$, $M_s = 5.0$.

Clearly, the strategy $M_a = 2.0$ and $M_s = 5.0$ is quite "dry" in that land is fallowed an average of 25 percent of the time. Not only is the relative frequency of fallow low, but land is not fallowed after barley when soil water is greater than 2.0 inches in the fall, and land is not fallowed after winter wheat when soil water in the spring is greater than 5 inches. It would appear that such a strategy could control saline seep problems once the soil profile below the root zone were dried out by a crop like alfalfa, or even safflower to some extent.

In cases where the farmer is unwilling to dry out the deep subsoil, this strategy would probably control percolation beyond the root zone, and ultimately control the saline seep problem. But the process would be slow, just as the problem evolved slowly, and the exacerbated damage to the soil around the seeps might be an important economic consideration.

Using a fixed rotation for comparison, fallowing without regard to soil water will result in summer fallow being carried out with initial soil water in the fall greater than two inches about 70 percent of the time, and in the spring, fallowing will take place with soil water greater than five inches about two-thirds of the time. Obviously, the saline seep hazards from deep percolation of water are much greater when soil water is not used as a decision criterion since fallowing is never carried out when soil water is greater than two and five inches in the fall and spring, respectively, under the strategy M_a = 2.0 and M_s = 5.0.

The best fixed rotation for saline seep control and relatively high returns is four years in length: fallow-wheat-barley-wheat. Fallow only occurs 25 percent of the time and expected annual net returns under $3 wheat are $31.17 compared to $34.50 for alternate crop-fallow. This is with all variable costs deducted, but not fixed costs. But the strategy M_a = 2.0, M_s = 5.0 has an average annual net return of $33.43 and also has a relative frequency of fallow equal to 25 percent. This strategy has an edge of $2.25 per acre annually as well as being more effective in saline seep control.

III. THE SPRING WHEAT REGION

The research reported in this section has application in the spring wheat areas of Northern Montana and North Dakota. The most important empirical information required for economic decisions on cropping strategies is a spring wheat yield response surface, where the explanatory variables are soil water, precipitation, and daily maximum temperatures. Such an equation, with sufficiently broad application, can be used as the basis for estimating yields on both sides of the trade-off equation (2.16) in Section II of this report.

The decision problems are much simpler in the spring wheat region compared to a winter wheat area such as was analyzed in Section II because significant decisions on cropping are made only in the spring. With a general wheat yield equation containing soil water at seeding, precipitation, and temperature variables which is applicable to a widely representative soil, optimal decision rules can be estimated across the various locations and climates of the spring wheat region. One piece of information needed to obtain an estimate of the decision rule is average soil water after a year of fallow (and after a year in crop, depending on the region); this information is substituted into the wheat yield equation as soil water at seeding to estimate the right hand side of (2.16). The other information required for an estimate of both sides of (2.16) is a fairly long set of time series data on precipitation and temperature measurements during the growing season. A series as short as ten years or even less could be useful, but 30 or 40 years would be desirable.

Before going into more detail on how the wheat yield response surface is utilized for developing economic decision rules, the empirically estimated relationship is presented.

The Spring Wheat Yield Equation

The data base for estimation of yield response was long-term experiments done cooperatively by the U.S.D.A. and state agricultural experiment stations at Mandan, North Dakota and Havre, Montana. The data for Montana came directly from (Army and Hide), while that from North Dakota was obtained from the files at the Mandan Experiment Station. Details of the rotations started by the Office of Dryland Agriculture, U.S.D.A. are given in (Chilcott). The rotations used for spring wheat were those usually coded MCA (continuous cropping) and MCD (alternate crop-fallow), see for example (Army and Hide). Soil moisture measurements to four feet were taken on these treatments by one-foot layers, while most of the other rotations which might have been of interest did not have soil moisture measurements taken.

A great deal of research effort was expended trying to utilize spring wheat rotation data from other locations, but to no avail. Soil water data was either inadequate or questions arose as to the validity of the yield data. At Newell, South Dakota, we could not get the weather data to correlate with crop yields and had to conclude that the weather data available was not what it was supposed to be.

Even at Mandan, North Dakota, where we used the data, there was no record of emergence dates for the crops which reduced the precision in estimation that would have been possible otherwise. We estimated emergence date at Mandan from the detailed information available at the Havre, Montana, location.

One serious problem at Havre was the variation in soil moisture measurement dates. Soil water was supposedly measured at emergence, but was actually done sometime afterwards and this variation was quite a problem. An evaporation-precipitation accounting model was constructed to adjust measured soil water to its equivalent at emergence. Obviously, the adjustment was subject to considerable error, but this was the best that could be done. Using different soil water measurement dates relative to emergence date was unacceptable because precipitation measured from emergence date would then be confounded with the soil water.

At Mandan, soil water was measured at seeding time with considerable variation; the time of measurement was usually after seeding. These measurements were adjusted to correspond with an estimated emergence date based on average time from seeding to emergence at Havre. Obviously, the soil moisture variables which had to be used in the yield response analysis were subject to considerable error, especially when considered as separate variables for each one-foot layer, as contrasted to a single variable for the entire four feet.

Evidence which evolved in fitting separate equations to the two locations suggested that soil moisture variables were less dependable at Mandan than at Havre. The most likely explanation for this difference in reliability is the need at Mandan to estimate emergence date from seeding date, while emergence date was recorded at Havre.

The yield response analysis used emergence date as the reference point from which weekly periods were defined during the growing season. Precipitation and temperature variables were defined for each of 13 weeks measured from emergence date. Errors in estimated emergence dates tend to confound net effects of precipitation and termperature among the various weekly periods of phenological development of spring wheat. In addition, early season precipitation variables are confounded with the soil moisture measurements.

In spite of these serious problems with the data, relatively good statistical results were obtained for the pooled data at Havre and Mandan, or at least, the statistical results appeared to be quite satisfactory. Of course, the data problems should not be ignored in interpreting the results because the consequences of errors in variables are not always apparent in sample results.

The algebraic form of the equation fitted to the data is nonlinear and quite complicated. A multiplicative error term comparable to the model described in Section II was specified for spring wheat. This form for the statistical error proved superior to the usual additive form. One source of nonlinearity stems from the multiplicative statistical error because the logarithmic transformation required to get an additive error requires taking the logarithm of mean yield. Unless mean yield is multiplicative in the independent variables, the logarithmic transformation implies nonlinearity in the parameters of the model.

A composite variable is used which is a linear function with respect to its parameters but contains some interaction terms which are nonlinear in the water and temperature variables. The variables are defined below:

W_1, W_2, W_3, W_4 are plant available soil water variables for the first through fourth foot of soil;

P_1, P_2, . . . P_{13} are weekly precipitation variables ordered from emergence date;

T_1, T_2, . . . T_{13} are weekly average daily maximum temperature variables ordered from emergence date;

$X_1 = W_1 + W_2 + P_1 + . . . P_6$;

$X_2 = T_7 + T_8 + T_9$;

$X_3 = X_1/X_2$;

$X_4 = (W_1 + ... + W_4 + P_1 + ... + P_{13})^{1/2}(T_1 + ... + T_{13})/13$

$$Z = \begin{cases} 0 \text{ if the location is Havre} \\ 1 \text{ if the location is Mandan} \end{cases}$$

The X-variables are interactions to allow a more complex relationship between water and temperature, and among water and temperature variables in different time periods within the growing season. In an intuitive sense, X_1 is an early-season water variable; X_2 is a critical-development-period temperature variable; X_3 is an interaction between X_1 and X_2; and X_4 is a total season interaction between water and temperature.

The composite variable which is specified to be a homogenous linear function of the above defined water and temperature variables is designated by the letter u, and the yield equation fitted to the data is

$$(3.1) \qquad y = a_i b(1 - e^{-u^c}) \qquad , \qquad i = 1, 2, 3$$

where a_1, a_2, a_3, b, and c are unknown parameters. The three parameters a_1, a_2, and a_3 adjust the asymptotic yield parameter b according to whether the crop was planted on fallow or not and according to location (Havre or

Mandan). The norm represented by b is taken as wheat planted on stubble at Havre and the other three parameters are proportionate adjustments in b. The definitions for these adjustments in terms of asymptotic yields are as follows:

b corresponds to Havre on stubble

a_1b corresponds to Havre on fallow

a_2b corresponds to Mandan on stubble

a_3b corresponds to Mandan on fallow.

The general shape of the function in (3.1) is concave downward if c is less than one, but if c is greater than one, the curve has an inflection point to the right of zero. Thus when c is greater than one, the curve is the classic sigmoid shape postulated in production economics.

Each variable which enters the composite variable u linearly also requires another parameter to be estimated. A straightforward count gives a total of 40 parameters to represent the yield equation; 5 for (3.1) when the adjustments to b are considered and 35 variables listed which enter u linearly. Clearly we could use some device to reduce the number of parameters. The method used is due to R. A. Fisher; a polynomial constraint with time as the argument is placed on the coefficients associated with each P_1, P_2, ... P_{13} and T_1, T_2, ... T_{13}.

Let t be a continuous variable associated with time during the growing season measured from emergence date, i.e., $t = 0$ at emergence date and $t = 13$ for the end of the 13th week after emergence. The parameter estimates for the linear coefficients on precipitation are denoted by d_1, d_2, ... d_{13}. Then these coefficients are forced to obey a polynomial as follows:

(3.2) $d_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 \quad , \quad t = 1, 2, \ldots 13.$

Consequently, the 13 parameters $d_1, \ldots d_{13}$ are completely determined by the three parameters α_0, α_1, and α_2 which reduces the number of parameters in the model by ten.

The same procedure is used to estimate the linear coefficients on temperature; let these parameters be denoted by $e_1, e_2, \ldots e_{13}$. The polynomial used to constrain these parameter estimates is

(3.3) $e_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 \quad , \quad t = 1, 2, \ldots 13$

which saves nine parameters in the statistical estimation. A cubic instead of a quadratic was required on temperature variables to provide adequate flexibility.

In total, the polynomial constraint technique saved 19 parameters, giving a total net number of parameters in the model equal to 21. After estimation of the polynomial parameters in (3.2) and (3.3), the individual weekly coefficients for precipitation and temperature can be calculated as well as statistical standard errors for them because the transformations involved are linear.

Numerical results for the nonlinear regression estimation are given in Table 14. The asymptotic yield parameter, denoted by b in (3.1), tended to increase without limit in the statistical fitting procedure. Therefore this parameter was arbitrarily set equal to 40 which gave about the same fit as very large numbers obtained from the fitting procedure. Apparently the data do not contain high enough yields to estimate this parameter.

Table 14

Statistical Results for Spring Wheat Yield Response

Description of Parameter or Variable	Regression Coefficient	Standard Error	t-ratio
Asymptotic yield	40	---	---
Adjustments in asymptotic yield			
Fallow at Havre	1.22	0.0721	3.04*
Stubble at Mandan	1.09	0.0892	1.06*
Fallow at Mandan	1.30	0.0928	3.21*
Exponent on composite variable u	0.973	0.114	0.24*
Soil Water Variables			
1st foot	.145	.0473	3.07
2nd foot	.185	.0422	4.38
3rd foot	.129	.0385	3.35
4th foot	.114	.0441	2.59
Precipitation Variables			
1st week	.168	.0453	3.71
2nd week	.164	.0433	3.79
3rd week	.158	.0414	3.82
4th week	.151	.0394	3.83
5th week	.143	.0374	3.82
6th week	.133	.0354	3.76
7th week	.123	.0334	3.68
8th week	.112	.0315	3.56
9th week	.100	.0300	3.33
10th week	.086	.0290	2.98
11th week	.072	.0288	2.49
12th week	.056	.0297	1.89
13th week	.040	.0318	1.25
Temperature Variables			
1st week	- .00161	.00111	-1.45
2nd week	- .00042	.00080	-0.53
3rd week	.00027	.00067	0.40
4th week	.00054	.00059	0.92
5th week	.00047	.00053	0.89
6th week	.00014	.00052	0.27
7th week	- .00035	.00059	-0.59
8th week	- .00094	.00069	-1.36
9th week	- .00153	.00076	-2.01
10th week	- .00205	.00074	-2.77
11th week	- .00241	.00063	-3.83
12th week	- .00253	.00063	-4.02
13th week	- .00233	.00113	-2.06

Table 14 (cont'd)

Description of Parameter or Variable	Regression Coefficient	Standard Error	t-ratio
Interaction Variables			
Early season water, X_1	- .357	.0773	-4.62
Critical season temperature, X_2	.0194	.00468	4.15
X_1/X_2	29.4	6.24	4.71
Total water-temperature, X_4	- .00658	.00198	-3.32
Constant for Mandan location, Z	.0903	.0276	3.27

*The null hypothesis associated with the t-statistic is that the parameter is equal to one (instead of zero in the other cases).

As a point of reference in the phenology of the crop, heading is about the middle of week number eight and the soft-dough stage is about the middle of week number twelve. However, the coefficients on the weekly precipitation and temperature variables cannot be interpreted in isolation from the interaction terms where these same variables appear.

Net effects of individual variables (partial derivatives) are given in Table 15 for all variables at their mean levels for the entire sample of data at both locations. The quite large detrimental effects for temperature during weeks 7, 8, and 9 stem from the choice of interaction terms, namely, the ratio of early season water to critical season temperature. Choice of the critical season for temperature was based on a simple comparison of periods around heading time, and there is no reason to expect that the net effects are actually as choppy as estimated in Table 15. Nevertheless, the net effects for temperature would appear quite plausible.

One might have expected net effects of precipitation to be relatively larger in weeks 6, 7, and 8 preceding heading, but apparently early season precipitation is the most important when recognition is made of the empirical distribution of precipitation for the region under study. The nearly zero and negative effects for precipitation in weeks 11 through 13 suggest that the crop cannot effectively use water that late in the season. However, these effects are increased by the total season water-termperature interaction when total water is relatively large; possibly the extra water delays maturity and thus makes water later in the season more productive.

Table 15

Individual Variable Net Effects on Yield With All Variables at Mean Levels

Description of Variable	Net Effect on Yield* (bushels per acre)
Soil Water	
1st foot	1.87
2nd foot	2.71
3rd foot	1.54
4th foot	1.23
Precipitation Variables	
1st week	2.35
2nd week	2.27
3rd week	2.14
4th week	2.00
5th week	1.83
6th week	1.62
7th week	1.42
8th week	1.19
9th week	0.56
10th week	0.27
11th week	-0.02
12th week	-0.35
13th week	-0.69
Temperature Variables	
1st week	-0.50
2nd week	-0.48
3rd week	-0.46
4th week	-0.46
5th week	-0.46
6th week	-0.47
7th week	-1.08
8th week	-1.09
9th week	-1.10
10th week	-0.51
11th week	-0.52
12th week	-0.52
13th week	-0.52

*Net effect on yield is technically the partial derivative of the yield equation with respect to the designated variable with all variables at their sample means.

Net effects for soil water by one-foot layers seems to make sense on a priori reasoning. The second foot has a relatively high density of roots and is protected from evaporation losses compared to the top foot. The 3rd and 4th feet are progressively less effective sources of water because of fewer roots and greater distance from the main plant body.

Supplementary statistical results are given in Table 16. The statistical error correlations among treatments are recognized in the statistical model and the reported correlations are maximum likelihood estimates. Note the high correlations between stubble and fallow plots at the same locations as would be expected. The model seemed to fit the fallow treatment at Havre the worst, as evidenced by the larger standard error of estimate and smaller Durbin-Watson statistic; the latter measures serial correlation in the time series of residuals from the regression equation for the specific treatment.

Estimation of Conditional Expected Yields in Relation to Soil Water

The empirical estimate of the Havre-Mandan yield equation given in Table 14 can be used together with a time series on climatic data at a given site to estimate expected yield of spring wheat as a function of plant available soil water at seeding time. This functional relationship will be referred to as simply "the mean yield function in soil moisture."

Table 16

Supplementary Statistical Results for Spring Wheat

	Treatments			
	Havre Stubble	Havre Fallow	Mandan Stubble	Mandan Fallow
Standard Error of Estimate*	.241	.304	.220	.237
Durbin-Watson Statistic	2.41	1.39	2.35	2.22
Statistical Error Correlations				
Havre Stubble	1.0	.571	- .041	- .114
Havre Fallow		1.0	.275	- .055
Mandan Stubble			1.0	.427
Mandan Fallow				1.0

Degrees of Freedom = 98

Coefficient of Multiple Determination (R^2) = .926

*Unit of measure is the natural logarithm of yield. Comparable measures in the original units of bushels per acre for Havre and Mandan are 3.26 and 3.80, respectively, at mean yields for each location. (The average standard errors across stubble and fallow were used to get these estimates.)

This functional relationship could be estimated with four variables, one each for the four layers of soil water by one-foot depths, but the economic analysis was not attempted with that much precision on soil water. The primary reason for not doing so was its impracticality in application to actual farm decisions; farmers would probably use a soil probe to estimate total water and the precision of such a procedure would not warrant differentiation of water among the one-foot soil layers. In addition, the economic analysis is much simpler and more easily presented if there is only one soil moisture variable.

At Havre and Mandan, data were available by one-foot depths for an extended number of years. Mean plant available water in each foot of soil was used as a relative weight to reduce the individual soil water regression coefficients in Table 14 to a single coefficient for soil water to four feet. The final coefficient is a weighted average of the one-foot coefficients, with mean soil water by one-foot depths serving as weights at a given location. At Dickenson, North Dakota and Sidney, Montana, the weighted coefficient at Mandan was used in the analysis. It was obvious from the Havre and Mandan work that the weighted average coefficient was not very sensitive to different locations.

Suppose that n years of climatic data are available at a given site, i.e., daily maximum temperature and precipitation. A specific emergence date is specified for the location from which 13 weekly temperature and precipitation variables are constructed for each of the n years of data. These weekly climatic variables are used to construct the interaction variables in the spring wheat yield equation (see the definition of variables just before (3.1)).

The next step in calculating the mean yield function in soil moisture is to estimate the most appropriate asymptotic yield, $a_j b$, to use in (3.1). The only choice at Mandan or Havre is between planting on stubble or fallow. The assumption was made that the fallow asymptotic yield parameter is most appropriate for planting on either fallow or stubble when nitrogen fertilization is practiced; adjustments are made in production costs for the difference in nitrogen applied to stubble versus fallow. The statistical work for the Highwood Bench data in Section II suggested that there exists a "pure fallow effect" in addition to the influence of soil water and plant available nitrogen, but we cannot separate such an effect out from the nitrogen effect in these data from Havre and Mandan because nitrogen fertilization was not a factor in the experiments. At Dickenson, North Dakota and Sidney, Montana, the fallow asymptotic yield parameter at Mandan was assumed to be applicable. It is noted that the difference in asymptotic yield estimated for planting on fallow at Mandan is only seven percent higher than at Havre, certainly not a very significant consideration.

The primary calculations to estimate the mean yield function in soil moisture are as follows. For a given level of plant available soil moisture, the yield is calculated from the estimated parameters of (3.1) as given in Table 14 (with selections and modifications described above) for each of the n years of climatic data available. Let us denote a given level of plant available soil water in the top four feet of soil by W. Then for a given value of W, we have calaculated n predicted spring wheat yields, one for each year of the time series. The simple average of those n yields is the estimated conditional mean yield, given soil water at planting time* equals W. The standard deviation of conditional mean yields, given W, can be calculated from the same n predicted yields by the usual formula from statistical theory.

These conditional mean yields and standard deviations are given in tabular form in the results reported below for each location. For example, the Havre results are given in Table 17. Conceptually, conditional mean yields and the standard deviation of yields are each a continuous function of plant available water at seeding time. If desired, a continuous function could be fitted to the tabular data by regression to get an estimate of the corresponding continuous functions, but the tabular approximations are adequate for our purposes.

Cropping Strategy Decision Models For Spring Wheat Areas

Two types of decision models were used to determine optimal cropping strategies for spring wheat in the Northern Great Plains. The more complex model is a mathematical model known as a Markov chain. A simpler, more intuitive model used was the "basic trade-off equation" (BTE) described earlier in the text.

Markov Chain Decision Model

The basic decision model to analyze the choice between whether to plant a spring grain crop or to fallow the land is mathematically described as a Markov chain. A complete explanation of the model in the context used here is given in (Burt and Allison, Burt and Stauber).

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*Plant available soil water at emergency is assumed to be approximately equal to the amount measured at seeding time.

Briefly, the problem is to make a decision concerning the use of land (crop or fallow) that will maximize present value of net returns per acre over a given planning horizon. In the model the decision regarding the utilization of a tract of land is made shortly before seeding time for spring wheat. At this decision point, a quantitative evaluation is made of the information concerning plant available moisture. This evaluation is used as the basis for making the decision on land use. That is, information regarding plant available moisture at seeding time is used as the basis for deciding whether to plant spring wheat or to leave the land fallow.

In our present situation we are concerned only with a model to determine whether the land should be fallowed or planted to spring wheat. Assuming that consecutive years of fallow are not permitted for a given tract of ground, a cropping strategy would be comprised of the following simple rule:

> If the land was in fallow last year, plant wheat this year. If the land was in crop last year, determine the amount of available soil water at planting time. If available soil water is M_s inches or greater plant wheat, but if available soil water is less than M_s inches, fallow the land.

The critical available soil moisture value M_s is simply a given number of inches of plant available water and it completely determines the decision rule. Conceptually, there is some optimal choice of M_s for a given location, price of wheat, costs of production, and technology of production.

In order to find the optimal value of M_s in a given situation, certain types of information must be available for specified levels of available soil moisture. Therefore, the possible values of plant available moisture are defined as discrete intervals which are designated by the midpoints of the intervals. For instance, plant available soil moisture between the values of 2.0 inches to 3.0 inches is designated by the midpoint of the interval, 2.5 inches. In the Markov chain model, a variable such as soil moisture which describes the condition of a decision process is called a state variable.

Two general types of empirical information are needed to estimate reliable cropping strategies for a given location. One necessary empirical relationship is an estimate of average crop yield in relation to available soil moisture at seeding time. That is, for a given crop, we need to know average yield over many years of climatic variation for a given level of available soil moisture at seeding time. The second type of empirical relationship needed is the probability distribution for available soil moisture changes from one season to the next under given land use alternatives. If both types of empirical information are available, the more complex Markov chain decision model can be applied. If the conditional expected yield relationship is available, the basic trade-off equation can be applied even though the required probability distribution on soil moisture is not available.

The necessary data were available to apply the Markov chain model at Havre, Montana, and Mandan, North Dakota. Probability distributions were not available at Sidney, Montana and Dickinson, North Dakota, so the BTE was used at these locations. The BTE was used at Havre and Mandan also to provide a comparison of results between the two decision models.

A summary of the empirical data used in the models and the economic results of the models are summarized below by locations.

Havre, Montana

The expected yield relationship for Havre was estimated by using the Havre-Mandan yield equation reported in Table 14 together with a series of weather data from Havre. The conditional yield estimates and their standard deviations are presented in Table 17.

Probability distributions for available soil moisture transitions from one season to the next under given land use alternatives were obtained from recorded soil moistures at Havre. A formal statement of the model used to relate soil moisture at period t to soil moisture at period t-1 is given by

$$M_t = f(M_{t-1}, LU_{t-1}, LU_t) \tag{3.4}$$

Literally this relationship states that available soil moisture at the end of the present time period (M_t) is a function of soil moisture at the end of the previous time period (M_{t-1}), land use during the previous time period (LU_{t-1}) and land use during the present time period (LU_t). Where crop (C) or fallow (F) are the land use alternatives, four situations exist; they can be enumerated as:

1. $M_t = f(M_{t-1}, F_{t-1}, F_t)$
2. $M_t = f(M_{t-1}, F_{t-1}, C_t)$
3. $M_t = f(M_{t-1}, C_{t-1}, C_t)$
4. $M_t = f(M_{t-1}, C_{t-1}, F_t)$

Linear regression techniques were used to estimate the relationship between M_t and M_{t-1} for situations 2, 3, and 4. No data were available to estimate the relationship depicted in situation 1. It was assumed that the estimates obtained for situation 4 would suffice for situation 1. This

Table 17

Conditional Expected Yields of Spring Wheat for Specified Levels Of Available Soil Moisture at Havre, Montana

Available Soil Moisture	Conditional Expected Yield	Standard Deviation
(inches)	(bu)	(bu)
.5	3.98	5.99
1.0	4.83	6.44
1.5	5.89	6.88
2.0	7.05	7.22
2.5	8.31	7.46
3.0	9.68	7.56
3.5	11.07	7.63
4.0	12.45	7.67
4.5	13.82	7.68
5.0	15.17	7.66
5.5	16.49	7.63
6.0	17.78	7.57
6.5	19.04	7.51
7.0	20.27	7.42
7.5	21.47	7.32
8.0	22.62	7.20

assumption depends on no significant difference in the distribution of M_{t-1} throughout the soil profile following fallow and crop. The validity of this assumption is questionable but lack of data at Havre or similar sites left no viable alternative.

The regression model to be estimated for a given situation is expressed as

$$M_t = \hat{\alpha} + \hat{\beta} M_{t-1} + e_t \qquad (3.5)$$

Regression estimates for situations where data were available are summarized in Table 18.

Table 18

Estimates of Parameters for Regression of Available Soil Water in Time t on Available Soil Water In Time t-1 at Havre, Montana

Land Use Situation	Estimates		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
FC	4.025	.3650	1.320
CC	5.608	.1271	1.150
CF	7.335	.2311	1.123

The immediately following discussion treats the estimated unknown parameters in the regression model as though they were the actual values because the decision model takes these sample estimates as the "true" values. Under the assumptions of the linear regression model, M_t is normally distributed with mean, $E(M_t) = \hat{\alpha} + \hat{\beta}M_{t-1}$ and variance, Var (M_t) =

$E(e_t^2) = \hat{\sigma}^2$. This information is used to derive the conditional probabilities needed in the Markov chain decision model. The procedure is sketched briefly here.

A given value of M_{t-1} implies the mean value of M_t, i.e., $E(M_t) = \hat{\alpha} + \hat{\beta}M_{t-1}$. Since M_t is distributed normally with standard deviation, σ, the probability of occurrence of any specified value for M_t can be found using the Z transformation,

$$\frac{M_t - E(M_t)}{\hat{\sigma}} = Z$$

and the standard normal tables.

Cost and Returns Data

The cost data for the land use alternatives, summer fallow or planting spring wheat, were taken from cost and returns studies for McCone and Sheridan counties (Schaefer, Stauber, and Luft, 1976a, 1976b).

The variable costs were adjusted to reflect lower levels of input use at unfavorable soil moisture levels. The details of the adjustments are not presented but in general the input reductions were assumed to occur in seed, nitrogen and phosphorus fertilizer, insecticide, miscellaneous expense and interest on operating capital. Crop insurance was omitted as a cost item because the FCIC does not issue loans on recrop acreages. Charges for operator labor were also omitted from the variable costs. The costs used at Havre are presented in Table 19.

The greater variable cost of spring wheat on stubble compared to fallow is mainly due to extra nitrogen fertilizer costs to compensate for lack of mineralized nitrogen under recrop conditions.

Table 19

Yield and Variable Costs by Specified Soil Moisture Levels For Recrop Spring Wheat, Spring Wheat on Fallow And Fallow at Havre, Montana

Soil Moisture	Yield	Variable Costs of Spring Wheat: Fallow	Variable Costs of Spring Wheat: Recrop	Variable Costs of Fallow
(inches)	(bushels)	(dollars)	(dollars)	(dollars)
.5	3.98	5.00	5.00	4.00
1.5	5.89	10.00	10.00	4.00
2.5	8.31	15.00	16.70	4.00
3.5	11.07	20.00	22.25	4.00
4.5	13.82	20.00	25.25	4.00
5.5	16.49	20.00	25.25	4.00
6.5	19.04	20.00	25.25	4.00
7.5	21.47	20.00	25.25	4.00

Gross returns and expected immediate returns per acre were calculated for three situations as enumerated below and presented in Table 20:

1. A wheat price of \$3.00 per bushel and 100 percent of estimated yields
2. A wheat price of \$3.00 per bushel and 120 percent of estimated yields
3. A wheat price of \$3.50 per bushel and 120 percent of estimated yields

Economic Results

The results of the Markov Chain Model are presented in Table 21 and graphically in Figure 1. The numerical results were derived by solving the system of simultaneous equations (Burt and Allison, p. 128) implied by selected values of M_s, the critical value for available soil moisture. Recall, that if available soil moisture is M_s inches or greater, wheat is planted. If available soil moisture is less than M_s, the land is fallowed. For example, $M_s = 0$ implies continuous cropping while $M_s = \infty$ implies an alternate crop-fallow rotation, if consecutive fallow is excluded on a given tract of land.

The net returns figures presented in Table 21 and Figure 1 are, more precisely, the amortized expected present value of net returns associated with an infinite planning horizon using an interest rate of 10 percent. The returns represent the estimated annual net returns above variable costs of production at the specified yield and price assumptions.

The optimal value of the decision variable M_s is seen to be 4 for

Table 20

Gross and Expected Immediate Returns for
Spring Wheat on Fallow and Recropped at Havre, Montana

Available Soil Moisture	Yield	Gross Returns			Expected Immediate Returns on Fallow			Expected Immediate Returns for Recrop		
		100%-$3.00	120%-$3.00	120%-$3.50	100%-$3.00	120%-$3.00	120%-$3.50	100%-$3.00	120%-$3.00	120%-$3.50
(inches)	(bushels)	(dollars)	(dollars)	(dollars)	(dollars)	(dollars)	(dollars)	(dollars)	(dollars)	(dollars)
.5	3.98	11.94	14.33	16.72	6.94	9.33	11.72	6.94	9.33	11.72
1.5	5.89	17.67	21.20	24.74	7.67	11.20	14.74	7.67	11.20	14.74
2.5	8.31	24.93	29.92	34.90	9.93	14.92	19.90	8.23	13.22	18.20
3.5	11.07	33.21	39.85	46.49	13.21	19.85	26.49	10.96	17.60	24.24
4.5	13.82	41.46	49.75	58.04	21.46	29.75	38.04	16.21	24.50	32.79
5.5	16.49	49.47	59.36	69.26	29.47	39.36	49.26	24.22	34.11	44.01
6.5	19.04	57.12	68.54	79.97	37.12	48.54	59.97	31.87	43.29	54.72
7.5	21.47	64.41	77.29	90.17	44.41	57.29	70.17	39.16	52.04	64.92

Table 21

Average Annual Net Returns in Relation to the Decision Rule Variable, M_s for Three Specified Yield-Price Situations, Havre, Montana

Decision Rule Variable, M_s	Average Annual Net Returns**		
	100%-$3.00	120%-$3.00	120%-$3.50
(inches available soil water)		(dollars)	
0	10.31	15.84	21.38
1	10.74	16.51	22.29
2	11.52	17.51	23.49
3	12.49	18.43	24.40*
4	12.97*	18.66*	24.35
5	12.84	18.22	23.60
6	12.42	17.57	22.73
7	12.19	17.26	22.32
8	12.13	17.17	22.21

*Optimum net returns per acre.

**A 10 percent interest rate was used for amortization.

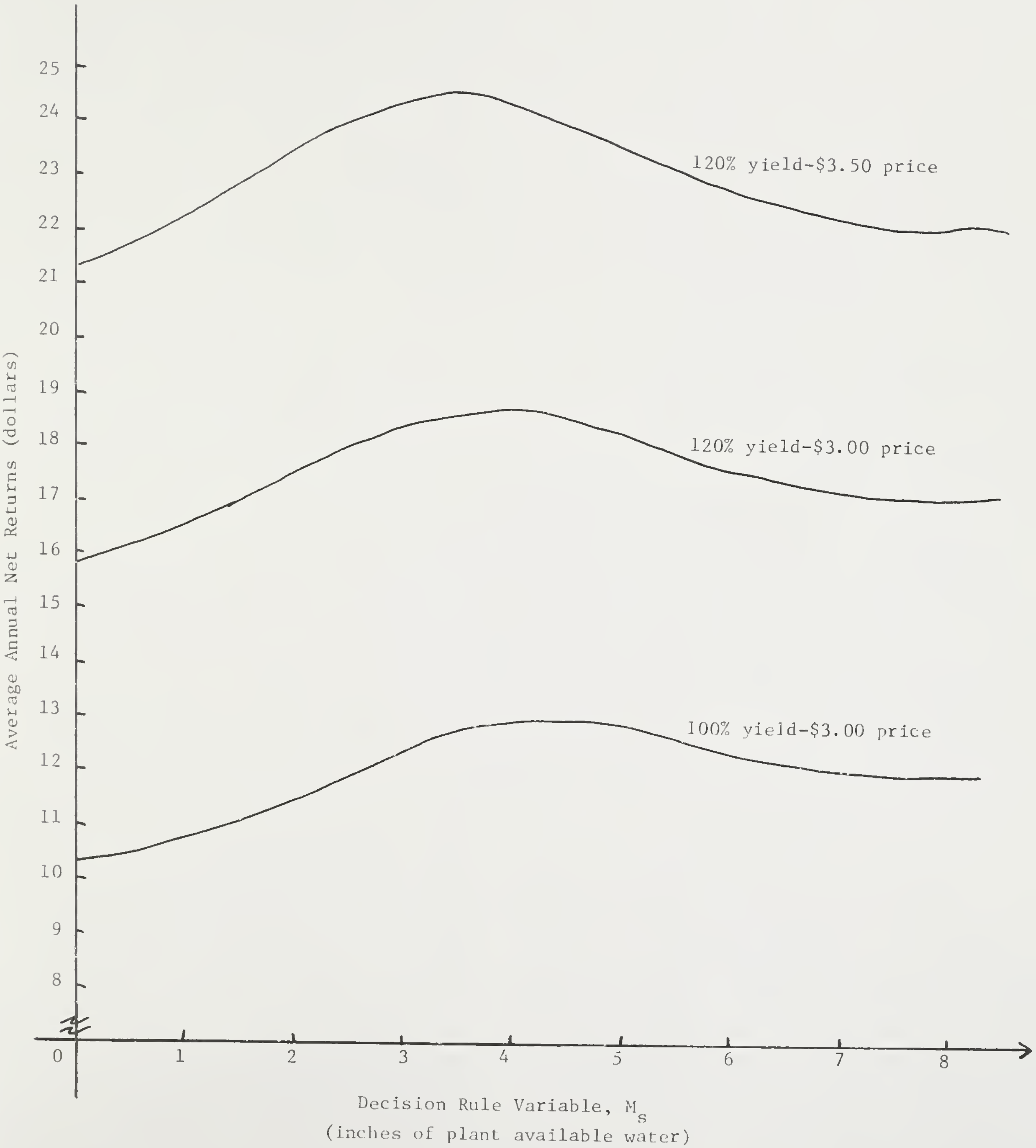

Figure 1 Average Annual Net Returns in Relation to the Decision Variable, M_s, at Havre, Montana

both of the situations where price of wheat is equal to \$3.00. The optimal value of the decision variable is $M_s = 3$ for the 120% yield-\$3.50 price situation. In all cases, alternate crop-fallow ($M_s = \infty$) is economically superior to continuous cropping ($M_s = 0$). The superiority of alternate crop-fallow over continuous cropping is inversely related to the magnitude of net returns as affected by yield and price assumptions.

In terms of saline seep abatement, it can be seen that the grain grower could move to a strategy which would result in less frequent fallow without a great loss in economic returns. In the 100% yield-\$3.00 price situation, fallowing if available soil water is 3 inches or less rather than at 4 inches would reduce annual average net returns by \$.48 per acre. Lowering the decision rule value of M_s from 3 inches to 2 inches however would result in an additional loss of \$.97 per acre. It should be noted that this loss would occur on all crop acres in the farming unit and is not a trivial loss. A loss of \$.97 per acre is approximately 7.5 percent of the net return under an optimal selection of M_s. (The percentage loss is substantially more if fixed costs are taken into account.) The farm operator must weigh the direct economic losses resulting from deviation from the optimal decision rule value for M_s against the potential benefits realized from saline seep control.

If the decision maker is willing to sacrifice economic returns, in order to control saline seep, adjustment of the value of the decision variable M_s to 3 inches has obvious advantages. This adjustment will cost him \$.48 per acre in net returns but it will result in keeping the soil in a considerably drier condition. It can be shown that the expected amount of available water in the soil profile prior to a year of fallow would be reduced by approximately .6 of an inch by adjusting the value of M_s downward from 4 inches to 3 inches. In order to illustrate the advantage of the decision rule ($M_s = 3$) over alternate crop-fallow, the probability of fallowing when available soil water exceeds 3 inches in the crop-fallow rotation must be known. The probability distribution of soil water after a year of crop when following a rotation of alternate crop-fallow is given in Table 22.

The level of M_s used in the decision rule is the upper limit on soil water at which fallow is allowed. That is, if the rule $M_s = 3$ is followed the land will never be fallowed when available soil moisture exceeds 3 inches at seeding time for spring wheat. Examination of Table 22 reveals the saline seep abatement advantages of the rule $M_s = 3$. Under an alternate crop-fallow system the soil would be fallowed when available soil moisture was greater than 3 inches 64 percent of the time. The reduction in deep percolation losses of soil water under the decision rule $M_s = 3$, as contrasted to alternate crop-fallow, is apparent.

It can be shown that the decision rule $M_s = 3$ will result in the farm being in crop approximately two-thirds of the time and in fallow one-third of the time. Under this rule, the land is recropped approximately one-half of the time. That is, approximately one-half of the years after a crop is grown the land will be recropped and approximately one-half of the years after a crop is grown the land will be fallowed. Under the assumptions of the model, a crop will always be planted after a year of fallow.

It is important to recognize that not only is the frequency of crop

Table 22

Probability Distribution of Available Soil Water After a Year of Crop for Alternate Crop-Fallow at Havre, Montana

Available Soil Water	Probability	Cumulative Probability
(inches)		
0.0 - 1.0	.04	.04
1.0 - 2.0	.10	.14
2.0 - 3.0	.22	.36
3.0 - 4.0	.28	.64
4.0 - 5.0	.22	.86
5.0 - 6.0	.10	.96
6.0 - 7.0	.03	.99
7.0 - 8.0	.01	1.00

higher than under an alternate crop-fallow rotation, but also, the land is in crop during seasons in which available soil water at seeding is relatively high. This fact is particularly important for saline seep control. A three-year rotation with fallow followed by two years of crop would also result in a frequency of crop of two years out of three. But this fixed rotation would be inferior to a decision rule given by $M_s = 3$ for purpose of saline seep control. Soil moisture would never exceed three inches prior to a year of fallow under the rule given by $M_s = 3$. This certainly would not be true for the fixed rotation.

Basic Trade-off Equation

An application of the basic trade-off equation (BTE) was made at Havre to see if it yields results consistent with the Markov chain model. The BTE is stated as:

(net return from the seeded crop in relation to available soil moisture)
= (average net returns from a crop after a year of fallow)
- (costs of the fallow)
- (average annual net returns under the best fixed crop rotation)

At Havre the best fixed crop rotation is assumed to be alternate crop-fallow (as opposed to continuous crop), and average available soil water is 5.59 inches after fallow and 3.12 inches after crop. Referring to the expected conditional yield information for Havre and the costs and returns information presented in Tables 19 and 20 the information needed in the BTE can be summarized:

average yield after fallow = 16.72 bushels
variable cost of fallow = $4.00
variable cost after fallow = $20.00

Using a \$3.00 price for wheat, the right hand side of the BTE becomes

(\$3.00 x 16.72 - \$20.00) - (\$4.00) - [(\$3.00 x 16.72 - \$20.00 - \$4.00)/2]

= \$30.16 - \$4.00 - \$13.08

= \$13.08

Application of the BTE implies that if the net return from the seeded crop in relation to available soil moisture is equal to or greater than \$13.08, the crop should be planted. Using the information concerning expected immediate returns in Table 20 it can be seen that if the decision is to be made following crop, the available soil moisture value which gives an average return of \$13.08 lies between 3.5 and 4.5 inches. Interpolation yields a value of 3.90 inches.

If the decision is to be made following fallow, the available soil moisture value which gives an average return of \$13.08 lies between 2.5 and 3.5 inches. Interpolation results in a value of 3.46 inches. These results agree closely with the results of the Markov chain model which indicates that the optimal land use alternative is fallow if the available soil moisture level after crop is less than 4 inches and plant wheat if the available soil moisture is equal to or greater than 4 inches.

Mandan, North Dakota

The expected yield relationship for Mandan was estimated by using the Havre-Mandan yield equation reported in Table 14 together with a series of weather data from Mandan. The expected yield estimates and their standard deviations are presented in Table 23.

Probability distributions for availale soil moisture changes from one season to the next under given land use alternatives were estimated from

Table 23

Conditional Expected Yields of Spring Wheat For Specified Levels of Available Soil Moisture at Mandan, North Dakota

Available Soil Moisture	Conditional Expected Yield	Standard Deviation
(inches)	(bu)	(bu)
.5	8.26	8.69
1.0	9.43	8.90
1.5	10.70	9.04
2.0	12.05	9.11
2.5	13.46	9.07
3.0	14.87	9.02
3.5	16.25	8.96
4.0	17.60	8.91
4.5	18.92	8.84
5.0	20.21	8.77
5.5	21.47	8.67
6.0	22.71	8.55
6.5	23.90	8.43
7.0	25.06	8.29
7.5	26.19	8.15
8.0	27.27	8.00
8.5	28.32	7.84
9.0	29.33	7.68

recorded soil moisture at Mandan. The procedures used at Mandan were identical to those used at Havre which have been explained earlier in the report. The regression estimates for Mandan are summarized in Table 24.

Again, there was no soil moisture series available to relate the effect of fallow on soil moisture measured after a year of fallow. The estimates for the effect of fallow on soil moisture after a year of crop were used in this instance.

Costs and Returns Data

The cost data derived at Havre was used for Mandan without adjustment. Gross returns and expected immediate returns were derived for a 100% yield-$3.00 wheat price and a 120% yield-$3.50 wheat price. The returns data are presented in Table 25.

Economic Results

The results of the Markov Chain Model are presented in Table 26 and graphically in Figure 2. The optimal value of the decision variable, M_s, is zero which implies continuous cropping. It is obvious that there is no economic loss from continuous cropping and fallow should not be practiced for purposes of soil water management even if saline seep is not a problem. It is also clear that the absolute difference in average net returns (which do not consider fixed costs) between continuous crop and alternate crop-fallow would more than compensate for the savings in fixed costs associated with a crop-fallow practice.

Basic Trade-off Equation

Application of the basic trade-off equation (BTE) at Mandan yields results consistent with the Markov chain model.

Table 24

Estimates of Parameters for Regression of Available Soil Water in Time t on Available Soil Water In Time t-1 at Mandan, North Dakota

Land Use Situation	Estimates		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$
FC	0.84	0.670	1.602
CC	2.54	0.424	1.421
CF	4.16	0.418	1.315

Table 25

Gross and Expected Immediate Returns for Spring Wheat On Fallow and Recropped at Mandan, North Dakota

Soil Moisture	Yield	Gross Returns		Expected Immediate Returns			
				Fallow		Recrop	
		100%-$3.00	120%-$3.50	100%-$3.00	120%-$3.50	100%-$3.00	120%-$3.50
(inches)	(bushels)			(dollars)			
.5	8.26	24.78	34.69	19.78	29.69	19.78	26.69
1.5	10.70	32.10	44.94	22.10	34.94	22.10	34.94
2.5	13.47	40.41	56.57	25.41	41.57	23.71	39.87
3.5	16.25	48.75	68.25	28.75	48.25	26.50	46.00
4.5	18.92	56.76	79.46	36.76	59.46	31.51	54.21
5.5	21.47	64.41	90.17	44.41	70.17	39.16	64.92
6.5	23.90	71.70	100.38	51.70	80.38	46.45	75.12
7.5	26.19	78.57	110.00	58.70	90.00	53.32	84.75
8.5	28.32	84.96	118.94	64.96	98.94	59.71	93.69

Table 26

Average Annual Net Returns in Relation to the Decision Rule Variable, M_s for Two Specified Yield-Price Situations, Mandan, North Dakota

Decision Rule Variable, M_s	Average Annual Net Returns	
	100%-\$3.00	120%-\$3.50
(inches available soil water)	(dollars)	
0	36.40	60.87
1	36.22	60.62
2	35.71	59.81
3	34.70	57.90
4	33.23	54.82
5	31.28	50.91
6	28.97	46.80
7	27.00	43.89
8	25.72	41.42
9	24.85	40.05

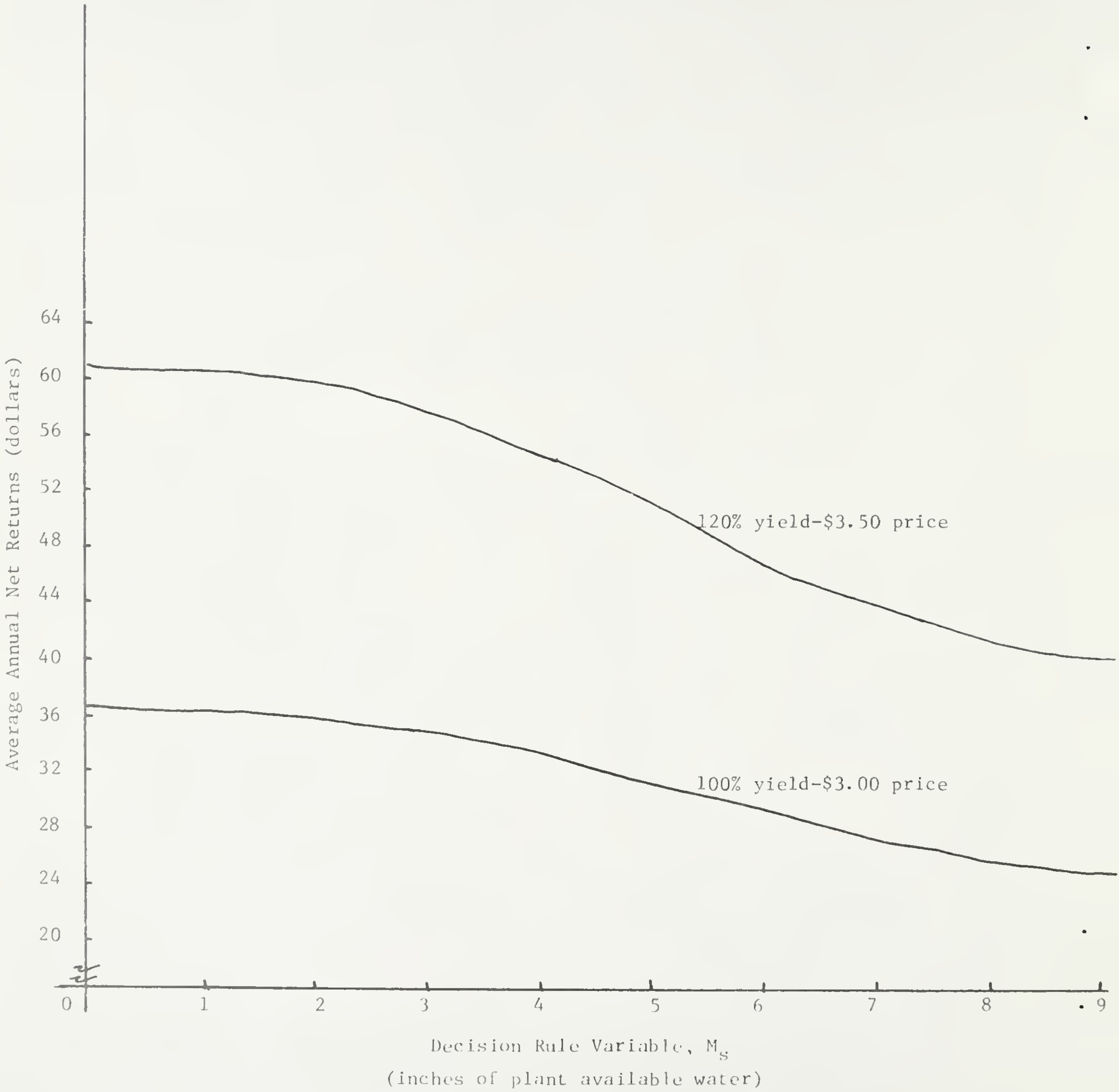

Figure 2. Average Annual Net Returns in Relation to the Decision Variable, M_s, at Mandan, North Dakota

At Mandan the best fixed crop rotation is continuous spring wheat and average available soil water is 6.22 inches after fallow and 4.45 inches after crop. Referring to the expected conditional yield information for Mandan and the costs and returns information presented in Tables 23 and 25 the information needed in the BTE can be summarized:

average yield after fallow = 23.22 bushels

average yield after crop = 18.79 bushels

variable cost of fallow = $4.00

variable cost after fallow = $20.00

variable costs after a crop = $25.25

Assuming a $3.00 price for wheat the right hand side of the BTE becomes

($3.00 x 23.22 - $20.00) - ($4.00) - ($3.00 x 18.79 - $25.25)

= ($69.69 - $20.00) - ($4.00) - ($56.37 - $25.25)

= $49.69 - $4.00 - $31.12

= $14.57

This implies that if the net returns from the seeded crop in relation to available soil moisture is equal to or greater than $14.57 that wheat should be planted. Using the information in Table 25 it can be determined that if the decision is to be made following a year of crop, the available soil moisture value that implies a return of $14.57 falls below the lowest possible soil moisture level. Therefore, the BTE yields the same results as the Markov chain model.

Sidney, Montana and Dickinson, North Dakota

Weather series were available at Sidney, Montana and Dickinson, North Dakota and conditional expected yields were calculated at these locations. This information is presented in Table 27.

Table 27

Conditional Expected Yields of Spring Wheat for Specified Levels of Available Soil Moisture at Sidney, Montana and Dickinson, North Dakota

Available Soil Moisture	Conditional Expected Yield	
	Sidney	Dickinson
(inches)	(bu)	(bu)
.5	8.02	10.44
1.0	9.28	11.94
1.5	10.61	13.44
2.0	11.97	14.92
2.5	13.34	16.38
3.0	14.73	17.81
3.5	16.13	19.22
4.0	17.52	20.59
4.5	18.88	21.92
5.0	20.21	23.21
5.5	21.51	24.46
6.0	22.72	25.67
6.5	24.00	26.84
7.0	25.19	27.96
7.5	26.35	29.04
8.0	27.46	30.08
8.5	28.54	31.08
9.0	29.58	32.04
9.5	30.57	32.95
10.0	31.53	33.83
10.5	32.45	34.67
11.0	33.34	35.47
11.5	34.18	36.24
12.0	34.99	36.97
12.5	35.77	37.66
13.0	36.51	38.33

Probability distributions for soil moisture changes from one season to the next under alternative land use decisions were not computed at these locations because of data limitations. Therefore it was not possible to apply the Markov chain model. However, the BTE was used to determine the value of the decision rule variable M_s. A \$3.00 price for wheat combined with the cost estimates used at Havre and Mandan, resulted in a value for M_s which was less than the lowest possible soil moisture level at both locations. This implies that continuous cropping would be the most economic cropping strategy at both locations and that if saline seep control were an objective, fallow should be diligently avoided.

References

Army, T. J. and J. C. Hide, Data from the Dryland Rotation and Tillage Studies at Huntley, Havre, and Moccasin, Montana, Plant and Soil Science Department. Mimeo Circular 43, Montana Agricultural Experiment Station, Bozeman, Montana, June 1962.

Burt, Oscar R. and John R. Allison, "Farm Management Decisions with Dynamic Programming," Journal of Farm Economics, Vol. 45 (Feb. 1963), pp. 121-136.

Burt, Oscar R. and Ralph D. Johnson, "Strategies for Wheat Production in the Great Plains," Journal of Farm Economics, Vol. 49 (Nov. 1967), pp. 881-899.

Burt, Oscar R. and M. S. Stauber, "Dryland Cropping Strategies to Prevent Saline Seep," Proceedings of Saline Seep Symposium. Montana State University, Dec. 1975.

Chilcott, E. C. The Relations Between Crop Yields and Precipitation in the Great Plains Area, Supplement 1--Crop Rotations and Tillage Methods, U.S.D.A., Misc. Circular No. 81, Washington, D.C., October 1931.

Fisher, R. A., "The Influence of Rainfall on the Yield of Wheat at Rothamsted," Royal Society (London), Philosophical Transactions, Series B, 213: pp. 89-142 (1924).

Kemeny, J. G., J. L. Snell, and G. L. Thompson, Introduction to Finite Mathematics, Reading, Mass.: Addison-Wesley, 1959.

Malinvaud, E. Statistical Methods of Econometrics, 2nd rev. ed., American-Elsevier, 1970.

Schaefer, Jerry, M. S. Stauber, LeRoy D. Luft, Enterprise Costs for Fallow, Spring wheat, Recrop Winter Wheat and Recrop Barley in Sheridan and Daniels Counties, Bull. 1133, Coop Extension Service, Montana State University, Bozeman, MT January, 1976a.

Schaefer, Jerry, M. S. Stauber, LeRoy D. Luft, Enterprise Costs for Fallow, Winter Wheat, Spring Wheat and Recrop Barley in McCone County, Bull. 1134, Coop Extension Service, Montana State University, Bozeman, MT February 1976b.

Schaefer, Jerry, Leroy D. Luft, and M. S. Stauber, Enterprise Costs for Fallow, Winter Wheat, and Barley in Chouteau County, Bull. 1140, Coop Extension Service, Montana State University, Bozeman, MT March 1976c.

APPENDIX

The wheat yield response surface and profit function for wheat and nitrogen prices at $3 per bushel and $0.20 per pound, respectively, are given here in tabular form. In addition, the slopes of the response surface with respect to each water and nitrogen, while holding the other variable constant, are given in tabular form. (Technically these slopes are partial derivatives of the yield equation with respect to each water and nitrogen.)

The table headings are self-explanatory except for columns four and five where the abbreviated headings are as follows:

DYDN = slope of the yield response surface with respect to nitrogen variation while holding water at the fixed level in column one and nitrogen at the level given in column two.

DYDW = slope of the yield response surface with respect to water variation while holding nitrogen at the fixed level in column two and water at the level given in column one.

Appendix Table 1

Winter Wheat Yield Response Surface In Tabular Form

WATER	NITROGEN	YIELD	DYDW	DYDN	PROFIT
5.0	.0	12.8	2.32	-.025	38.28
5.0	5.0	12.6	2.68	-.025	36.90
5.0	10.0	12.5	3.02	-.025	35.52
5.0	15.0	12.4	3.34	-.025	34.15
5.0	20.0	12.3	3.65	-.025	32.77
5.0	25.0	12.1	3.93	-.025	31.39
5.0	30.0	12.0	4.21	-.025	30.01
5.0	35.0	11.9	4.46	-.025	28.63
5.0	40.0	11.8	4.70	-.025	27.26
5.0	45.0	11.6	4.92	-.025	25.88
5.0	50.0	11.5	5.12	-.025	24.50
5.0	55.0	11.4	5.31	-.025	23.12
5.0	60.0	11.2	5.48	-.025	21.75
5.0	65.0	11.1	5.64	-.025	20.37
5.0	70.0	11.0	5.77	-.025	18.99
5.0	75.0	10.9	5.89	-.025	17.61
5.0	80.0	10.7	6.00	-.025	16.23
5.0	85.0	10.6	6.09	-.025	14.86
5.0	90.0	10.5	6.16	-.025	13.48
6.0	.0	14.9	2.04	.047	44.83
6.0	5.0	15.2	2.39	.043	44.50
6.0	10.0	15.4	2.72	.040	44.13
6.0	15.0	15.6	3.03	.036	43.70
6.0	20.0	15.7	3.32	.033	43.22

```
WATER      NITROGEN     YIELD       DYDW        DYDN      PROFIT
6.0          25.0        15.9       3.60        .030       42.69
6.0          30.0        16.0       3.86        .026       42.11
6.0          35.0        16.2       4.10        .023       41.48
6.0          40.0        16.3       4.33        .020       40.80
6.0          45.0        16.4       4.54        .016       40.07
6.0          50.0        16.4       4.74        .013       39.29
6.0          55.0        16.5       4.91        .010       38.46
6.0          60.0        16.5       5.07        .006       37.58
6.0          65.0        16.5       5.21        .003       36.65
6.0          70.0        16.6       5.34       -.001       35.66
6.0          75.0        16.5       5.45       -.004       34.63
6.0          80.0        16.5       5.54       -.007       33.55
6.0          85.0        16.5       5.62       -.011       32.41
6.0          90.0        16.4       5.68       -.014       31.23
7.0            .0        16.8       1.76        .116       50.54
7.0           5.0        17.4       2.10        .109       51.23
7.0          10.0        17.9       2.42        .103       51.82
7.0          15.0        18.4       2.72        .096       52.31
7.0          20.0        18.9       3.00        .089       52.70
7.0          25.0        19.3       3.27        .082       52.99
7.0          30.0        19.7       3.51        .076       53.18
7.0          35.0        20.1       3.75        .069       53.26
7.0          40.0        20.4       3.96        .062       53.25
7.0          45.0        20.7       4.16        .056       53.13
7.0          50.0        21.0       4.35        .049       52.91
7.0          55.0        21.2       4.51        .042       52.59
7.0          60.0        21.4       4.66        .035       52.18
7.0          65.0        21.6       4.79        .029       51.66
7.0          70.0        21.7       4.91        .022       51.03
7.0          75.0        21.8       5.01        .015       50.31
7.0          80.0        21.8       5.09        .008       49.49
7.0          85.0        21.9       5.15        .002       48.57
7.0          90.0        21.8       5.20       -.005       47.54
7.0          95.0        21.8       5.23       -.012       46.42
7.0         100.0        21.7       5.25       -.018       45.19
7.0         105.0        21.6       5.24       -.025       43.87
7.0         110.0        21.5       5.22       -.032       42.44
7.0         115.0        21.3       5.19       -.039       40.91
8.0            .0        18.5       1.48        .183       55.42
8.0           5.0        19.4       1.81        .173       57.09
8.0          10.0        20.2       2.11        .163       58.62
```

```
WATER      NITROGEN     YIELD      DYDW       DYDN       PROFIT
8.0         15.0        21.0       2.40       .153       59.99
8.0         20.0        21.7       2.67       .143       61.21
8.0         25.0        22.4       2.93       .133       62.28
8.0         30.0        23.1       3.17       .123       63.20
8.0         35.0        23.7       3.39       .113       63.97
8.0         40.0        24.2       3.60       .103       64.59
8.0         45.0        24.7       3.78       .093       65.05
8.0         50.0        25.1       3.96       .083       65.36
8.0         55.0        25.5       4.11       .072       65.53
8.0         60.0        25.8       4.25       .062       65.54
8.0         65.0        26.1       4.37       .052       65.40
8.0         70.0        26.4       4.47       .042       65.11
8.0         75.0        26.6       4.56       .032       64.66
8.0         80.0        26.7       4.63       .022       64.07
8.0         85.0        26.8       4.69       .012       63.32
8.0         90.0        26.8       4.72       .002       62.43
8.0         95.0        26.8       4.74      -.008       61.38
8.0        100.0        26.7       4.75      -.018       60.18
8.0        105.0        26.6       4.73      -.028       58.83
8.0        110.0        26.4       4.70      -.039       57.33
8.0        115.0        26.2       4.66      -.049       55.68
9.0           .0        19.8       1.20       .249       59.45
9.0          5.0        21.0       1.52       .235       62.08
9.0         10.0        22.2       1.81       .222       64.51
9.0         15.0        23.2       2.09       .208       66.73
9.0         20.0        24.3       2.35       .195       68.75
9.0         25.0        25.2       2.60       .181       70.57
9.0         30.0        26.1       2.82       .168       72.19
9.0         35.0        26.9       3.03       .154       73.61
9.0         40.0        27.6       3.23       .141       74.82
9.0         45.0        28.3       3.41       .128       75.84
9.0         50.0        28.9       3.57       .114       76.65
9.0         55.0        29.4       3.71       .101       77.26
9.0         60.0        29.9       3.84       .087       77.67
9.0         65.0        30.3       3.95       .074       77.87
9.0         70.0        30.6       4.04       .060       77.88
9.0         75.0        30.9       4.12       .047       77.68
9.0         80.0        31.1       4.18       .033       77.28
9.0         85.0        31.2       4.22       .020       76.68
9.0         90.0        31.3       4.25       .006       75.88
9.0         95.0        31.3       4.25      -.007       74.88
```

```
 WATER    NITROGEN    YIELD     DYDW     DYDN     PROFIT

   9.0     100.0      31.2      4.25    -.020     73.67
   9.0     105.0      31.1      4.22    -.034     72.26
   9.0     110.0      30.9      4.18    -.047     70.65
   9.0     115.0      30.6      4.12    -.061     68.84
  10.0        .0      20.9       .93     .312     62.65
  10.0       5.0      22.4      1.23     .295     66.19
  10.0      10.0      23.8      1.51     .278     69.49
  10.0      15.0      25.2      1.78     .261     72.53
  10.0      20.0      26.4      2.03     .244     75.32
  10.0      25.0      27.6      2.26     .227     77.86
  10.0      30.0      28.7      2.48     .211     80.14
  10.0      35.0      29.7      2.68     .194     82.18
  10.0      40.0      30.7      2.86     .177     83.96
  10.0      45.0      31.5      3.03     .160     85.49
  10.0      50.0      32.3      3.18     .143     86.76
  10.0      55.0      32.9      3.31     .127     87.79
  10.0      60.0      33.5      3.43     .110     88.56
  10.0      65.0      34.0      3.52     .093     89.08
  10.0      70.0      34.4      3.61     .076     89.35
  10.0      75.0      34.8      3.67     .059     89.36
  10.0      80.0      35.0      3.72     .043     89.13
  10.0      85.0      35.2      3.75     .026     88.64
  10.0      90.0      35.3      3.77     .009     87.90
  10.0      95.0      35.3      3.77    -.008     86.91
  10.0     100.0      35.2      3.75    -.025     85.66
  10.0     105.0      35.1      3.71    -.042     84.16
  10.0     110.0      34.8      3.66    -.058     82.42
  10.0     115.0      34.5      3.59    -.075     80.41
  11.0        .0      21.7       .65     .372     65.00
  11.0       5.0      23.5       .94     .352     69.44
  11.0      10.0      25.2      1.21     .332     73.57
  11.0      15.0      26.8      1.46     .312     77.39
  11.0      20.0      28.3      1.70     .292     80.92
  11.0      25.0      29.7      1.93     .271     84.14
  11.0      30.0      31.0      2.13     .251     87.06
  11.0      35.0      32.2      2.32     .231     89.68
  11.0      40.0      33.3      2.49     .211     91.99
  11.0      45.0      34.3      2.65     .191     94.00
  11.0      50.0      35.2      2.79     .171     95.71
  11.0      55.0      36.0      2.91     .150     97.12
  11.0      60.0      36.7      3.01     .130     98.22
```

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WATER      NITROGEN     YIELD      DYDW       DYDN      PROFIT
11.0        65.0        37.3       3.10       .110       99.02
11.0        70.0        37.8       3.17       .090       99.52
11.0        75.0        38.2       3.23       .070       99.71
11.0        80.0        38.5       3.27       .049       99.61
11.0        85.0        38.7       3.29       .029       99.20
11.0        90.0        38.8       3.29       .009       98.49
11.0        95.0        38.8       3.28     -.011        97.47
11.0       100.0        38.7       3.25     -.031        96.15
11.0       105.0        38.5       3.20     -.051        94.53
11.0       110.0        38.2       3.14     -.072        92.61
11.0       115.0        37.8       3.06     -.092        90.39
12.0          .0        22.2        .37       .431       66.52
12.0         5.0        24.3        .64       .407       71.81
12.0        10.0        26.2        .91       .384       76.74
12.0        15.0        28.1       1.15       .360       81.32
12.0        20.0        29.8       1.38       .337       85.54
12.0        25.0        31.5       1.59       .313       89.42
12.0        30.0        33.0       1.79       .290       92.94
12.0        35.0        34.4       1.96       .266       96.10
12.0        40.0        35.6       2.13       .242       98.92
12.0        45.0        36.8       2.27       .219      101.38
12.0        50.0        37.8       2.40       .195      103.49
12.0        55.0        38.7       2.51       .172      105.24
12.0        60.0        39.5       2.60       .148      106.64
12.0        65.0        40.2       2.68       .125      107.69
12.0        70.0        40.8       2.74       .101      108.39
12.0        75.0        41.2       2.78       .078      108.73
12.0        80.0        41.6       2.81       .054      108.72
12.0        85.0        41.8       2.82       .031      108.36
12.0        90.0        41.9       2.81       .007      107.64
12.0        95.0        41.9       2.79     -.016       106.57
12.0       100.0        41.7       2.75     -.040       105.15
12.0       105.0        41.5       2.69     -.063       103.37
12.0       110.0        41.1       2.62     -.087       101.24
12.0       115.0        40.6       2.53     -.111        98.76
13.0          .0        22.4        .09       .487       67.20
13.0         5.0        24.8        .35       .460       73.30
13.0        10.0        27.0        .61       .433       79.01
13.0        15.0        29.1        .84       .406       84.30
13.0        20.0        31.1       1.06       .380       89.20
13.0        25.0        32.9       1.26       .353       93.69
```

WATER	NITROGEN	YIELD	DYDW	DYDN	PROFIT
13.0	30.0	34.6	1.44	.326	97.78
13.0	35.0	36.2	1.61	.299	101.46
13.0	40.0	37.6	1.76	.272	104.74
13.0	45.0	38.9	1.89	.245	107.62
13.0	50.0	40.0	2.01	.218	110.10
13.0	55.0	41.1	2.11	.191	112.17
13.0	60.0	41.9	2.19	.164	113.83
13.0	65.0	42.7	2.26	.137	115.10
13.0	70.0	43.3	2.31	.111	115.96
13.0	75.0	43.8	2.34	.084	116.41
13.0	80.0	44.2	2.35	.057	116.47
13.0	85.0	44.4	2.35	.030	116.12
13.0	90.0	44.5	2.34	.003	115.36
13.0	95.0	44.4	2.30	-.024	114.21
13.0	100.0	44.2	2.25	-.051	112.64
13.0	105.0	43.9	2.18	-.078	110.68
13.0	110.0	43.4	2.10	-.105	108.31
13.0	115.0	42.8	1.99	-.132	105.54
14.0	.0	22.3	-.19	.541	67.04
14.0	5.0	25.0	.06	.511	73.93
14.0	10.0	27.5	.30	.481	80.37
14.0	15.0	29.8	.53	.451	86.35
14.0	20.0	32.0	.73	.420	91.88
14.0	25.0	34.0	.92	.390	96.96
14.0	30.0	35.9	1.10	.360	101.58
14.0	35.0	37.6	1.25	.329	105.75
14.0	40.0	39.2	1.39	.299	109.47
14.0	45.0	40.6	1.51	.269	112.73
14.0	50.0	41.8	1.62	.239	115.54
14.0	55.0	43.0	1.71	.208	117.89
14.0	60.0	43.9	1.78	.178	119.79
14.0	65.0	44.7	1.83	.148	121.23
14.0	70.0	45.4	1.87	.118	122.23
14.0	75.0	45.9	1.89	.087	122.76
14.0	80.0	46.3	1.90	.057	122.85
14.0	85.0	46.5	1.89	.027	122.48
14.0	90.0	46.6	1.86	-.003	121.65
14.0	95.0	46.5	1.81	-.034	120.37
14.0	100.0	46.2	1.75	-.064	118.64
14.0	105.0	45.8	1.67	-.094	116.46
14.0	110.0	45.3	1.57	-.124	113.82

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WATER    NITROGEN   YIELD      DYDW      DYDN      PROFIT
14.0      115.0      44.6      1.46     -.155      110.72
15.0         .0      22.0      -.47      .593       66.04
15.0        5.0      24.9      -.23      .560       73.69
15.0       10.0      27.6       .00      .526       80.83
15.0       15.0      30.2       .21      .492       87.47
15.0       20.0      32.5       .41      .459       93.60
15.0       25.0      34.7       .59      .425       99.23
15.0       30.0      36.8       .75      .391      104.35
15.0       35.0      38.7       .89      .358      108.97
15.0       40.0      40.4      1.02      .324      113.09
15.0       45.0      41.9      1.13      .291      116.70
15.0       50.0      43.3      1.23      .257      119.81
15.0       55.0      44.5      1.31      .223      122.41
15.0       60.0      45.5      1.37      .190      124.51
15.0       65.0      46.4      1.41      .156      126.10
15.0       70.0      47.1      1.44      .123      127.19
15.0       75.0      47.6      1.45      .089      127.78
15.0       80.0      48.0      1.44      .055      127.86
15.0       85.0      48.1      1.42      .022      127.44
15.0       90.0      48.2      1.38     -.012      126.51
15.0       95.0      48.0      1.32     -.046      125.08
15.0      100.0      47.7      1.25     -.079      123.14
15.0      105.0      47.2      1.16     -.113      120.70
15.0      110.0      46.6      1.05     -.146      117.76
15.0      115.0      45.8       .93     -.180      114.31
16.0         .0      21.4      -.75      .643       64.20
16.0        5.0      24.5      -.52      .606       72.57
16.0       10.0      27.5      -.30      .569       80.38
16.0       15.0      30.2      -.10      .532       87.64
16.0       20.0      32.8       .09      .495       94.34
16.0       25.0      35.2       .25      .458      100.49
16.0       30.0      37.4       .40      .421      106.08
16.0       35.0      39.4       .54      .384      111.12
16.0       40.0      41.2       .66      .347      115.61
16.0       45.0      42.8       .76      .310      119.54
16.0       50.0      44.3       .84      .273      122.91
16.0       55.0      45.6       .91      .236      125.73
16.0       60.0      46.7       .96      .199      127.99
16.0       65.0      47.6       .99      .162      129.70
16.0       70.0      48.3      1.01      .125      130.86
16.0       75.0      48.8      1.01      .088      131.46
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WATER      NITROGEN    YIELD       DYDW      DYDN      PROFIT
16.0        80.0       49.2        .99       .051      131.51
16.0        85.0       49.3        .95       .014      131.00
16.0        90.0       49.3        .90      -.023      129.93
16.0        95.0       49.1        .83      -.060      128.31
16.0       100.0       48.7        .75      -.097      126.14
16.0       105.0       48.1        .65      -.134      123.41
16.0       110.0       47.4        .53      -.171      120.13
16.0       115.0       46.4        .40      -.208      116.29
17.0        25.0       35.2       -.08       .489      100.75
17.0        30.0       37.6        .06       .448      106.78
17.0        35.0       39.7        .18       .408      112.20
17.0        40.0       41.7        .29       .368      117.02
17.0        45.0       43.4        .38       .327      121.24
17.0        50.0       44.9        .45       .287      124.84
17.0        55.0       46.3        .51       .247      127.85
17.0        60.0       47.4        .54       .206      130.25
17.0        65.0       48.3        .57       .166      132.04
17.0        70.0       49.1        .57       .126      133.23
17.0        75.0       49.6        .56       .085      133.81
17.0        80.0       49.9        .53       .045      133.79
17.0        85.0       50.1        .49       .005      133.16
17.0        90.0       50.0        .43      -.036      131.93
17.0        95.0       49.7        .35      -.076      130.09
17.0       100.0       49.2        .25      -.116      127.64
17.0       105.0       48.5        .14      -.157      124.59
17.0       110.0       47.6        .01      -.197      120.94
17.0       115.0       46.6       -.14      -.237      116.68
18.0        25.0       35.0       -.42       .517      100.00
18.0        30.0       37.5       -.29       .474      106.44
18.0        35.0       39.7       -.17       .430      112.21
18.0        40.0       41.8       -.08       .386      117.33
18.0        45.0       43.6       -.00       .343      121.80
18.0        50.0       45.2        .06       .299      125.61
18.0        55.0       46.6        .11       .255      128.76
18.0        60.0       47.8        .13       .211      131.26
18.0        65.0       48.7        .14       .168      133.10
18.0        70.0       49.4        .14       .124      134.29
18.0        75.0       49.9        .12       .080      134.82
18.0        80.0       50.2        .08       .037      134.70
18.0        85.0       50.3        .02      -.007      133.92
18.0        90.0       50.2       -.05      -.051      132.48
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WATER	NITROGEN	YIELD	DYDW	DYDN	PROFIT
18.0	95.0	49.8	-.14	-.095	130.39
18.0	100.0	49.2	-.25	-.138	127.65
18.0	105.0	48.4	-.37	-.182	124.24
18.0	110.0	47.4	-.51	-.226	120.19
18.0	115.0	46.2	-.67	-.269	115.47
19.0	25.0	34.4	-.75	.544	98.25
19.0	30.0	37.0	-.63	.497	105.06
19.0	35.0	39.4	-.53	.450	111.15
19.0	40.0	41.5	-.45	.402	116.54
19.0	45.0	43.4	-.38	.355	121.23
19.0	50.0	45.1	-.33	.308	125.21
19.0	55.0	46.5	-.30	.261	128.48
19.0	60.0	47.7	-.28	.214	131.04
19.0	65.0	48.6	-.28	.167	132.90
19.0	70.0	49.4	-.30	.120	134.06
19.0	75.0	49.8	-.33	.073	134.50
19.0	80.0	50.1	-.38	.026	134.25
19.0	85.0	50.1	-.45	-.021	133.28
19.0	90.0	49.9	-.53	-.068	131.61
19.0	95.0	49.4	-.63	-.115	129.23
19.0	100.0	48.7	-.75	-.162	126.15
19.0	105.0	47.8	-.88	-.209	122.36
19.0	110.0	46.6	-1.03	-.257	117.87
19.0	115.0	45.2	-1.20	-.304	112.67
20.0	25.0	33.5	-1.09	.568	95.50
20.0	30.0	36.2	-.98	.517	102.64
20.0	35.0	38.7	-.89	.467	109.02
20.0	40.0	40.9	-.81	.417	114.65
20.0	45.0	42.8	-.76	.366	119.52
20.0	50.0	44.5	-.72	.316	123.63
20.0	55.0	46.0	-.70	.265	126.99
20.0	60.0	47.2	-.69	.215	129.59
20.0	65.0	48.1	-.70	.164	131.43
20.0	70.0	48.8	-.73	.114	132.52
20.0	75.0	49.3	-.77	.064	132.85
20.0	80.0	49.5	-.83	.013	132.43
20.0	85.0	49.4	-.91	-.037	131.24
20.0	90.0	49.1	-1.01	-.088	129.31
20.0	95.0	48.5	-1.12	-.138	126.61
20.0	100.0	47.7	-1.25	-.189	123.16
20.0	105.0	46.7	-1.39	-.239	118.95

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 WATER    NITROGEN    YIELD     DYDW      DYDN     PROFIT
 20.0      110.0      45.3     -1.55     -.290     113.99
 20.0      115.0      43.8     -1.73     -.340     108.26
 21.0       25.0      32.2     -1.42      .590      91.74
 21.0       30.0      35.1     -1.32      .536      99.19
 21.0       35.0      37.6     -1.24      .482     105.82
 21.0       40.0      39.9     -1.18      .428     111.65
 21.0       45.0      41.9     -1.14      .375     116.68
 21.0       50.0      43.6     -1.11      .321     120.89
 21.0       55.0      45.1     -1.10      .267     124.30
 21.0       60.0      46.3     -1.10      .213     126.90
 21.0       65.0      47.2     -1.12      .159     128.70
 21.0       70.0      47.9     -1.16      .106     129.69
 21.0       75.0      48.3     -1.22      .052     129.87
 21.0       80.0      48.4     -1.29     -.002     129.24
 21.0       85.0      48.3     -1.38     -.056     127.81
 21.0       90.0      47.9     -1.48     -.110     125.57
 21.0       95.0      47.2     -1.61     -.163     122.52
 21.0      100.0      46.2     -1.75     -.217     118.67
 21.0      105.0      45.0     -1.90     -.271     114.01
 21.0      110.0      43.5     -2.08     -.325     108.54
 21.0      115.0      41.8     -2.27     -.379     102.26
 22.0       25.0      30.7     -1.75      .610      86.98
 22.0       30.0      33.6     -1.67      .552      94.70
 22.0       35.0      36.2     -1.60      .495     101.55
 22.0       40.0      38.5     -1.55      .438     107.55
 22.0       45.0      40.6     -1.52      .381     112.70
 22.0       50.0      42.3     -1.50      .324     116.98
 22.0       55.0      43.8     -1.50      .267     120.41
 22.0       60.0      45.0     -1.51      .209     122.98
 22.0       65.0      45.9     -1.55      .152     124.69
 22.0       70.0      46.5     -1.60      .095     125.55
 22.0       75.0      46.8     -1.66      .038     125.55
 22.0       80.0      46.9     -1.75     -.019     124.69
 22.0       85.0      46.7     -1.85     -.076     122.97
 22.0       90.0      46.1     -1.96     -.134     120.40
 22.0       95.0      45.3     -2.10     -.191     116.97
 22.0      100.0      44.2     -2.25     -.248     112.68
 22.0      105.0      42.8     -2.41     -.305     107.53
 22.0      110.0      41.2     -2.60     -.362     101.53
 22.0      115.0      39.2     -2.80     -.419      94.67
 23.0       25.0      28.7     -2.09      .627      81.22
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WATER	NITROGEN	YIELD	DYDW	DYDN	PROFIT
23.0	30.0	31.7	-2.02	.567	89.17
23.0	35.0	34.4	-1.96	.506	96.21
23.0	40.0	36.8	-1.92	.446	102.35
23.0	45.0	38.9	-1.89	.385	107.58
23.0	50.0	40.6	-1.89	.325	111.90
23.0	55.0	42.1	-1.90	.264	115.32
23.0	60.0	43.3	-1.92	.203	117.82
23.0	65.0	44.1	-1.97	.143	119.42
23.0	70.0	44.7	-2.03	.082	120.11
23.0	75.0	45.0	-2.11	.022	119.89
23.0	80.0	44.9	-2.20	-.039	118.77
23.0	85.0	44.6	-2.31	-.099	116.73
23.0	90.0	43.9	-2.44	-.160	113.79
23.0	95.0	43.0	-2.58	-.220	109.94
23.0	100.0	41.7	-2.75	-.281	105.19
23.0	105.0	40.2	-2.92	-.341	99.52
23.0	110.0	38.3	-3.12	-.402	92.95
23.0	115.0	36.2	-3.33	-.462	85.47
24.0	25.0	26.5	-2.42	.642	74.45
24.0	30.0	29.5	-2.36	.579	82.61
24.0	35.0	32.3	-2.31	.515	89.81
24.0	40.0	34.7	-2.29	.451	96.05
24.0	45.0	36.8	-2.27	.387	101.33
24.0	50.0	38.6	-2.28	.323	105.66
24.0	55.0	40.0	-2.30	.259	109.02
24.0	60.0	41.1	-2.34	.195	111.43
24.0	65.0	42.0	-2.39	.131	112.88
24.0	70.0	42.5	-2.46	.068	113.37
24.0	75.0	42.6	-2.55	.004	112.91
24.0	80.0	42.5	-2.66	-.060	111.48
24.0	85.0	42.0	-2.78	-.124	109.10
24.0	90.0	41.3	-2.92	-.188	105.76
24.0	95.0	40.2	-3.07	-.252	101.46
24.0	100.0	38.7	-3.25	-.316	96.20
24.0	105.0	37.0	-3.43	-.380	89.99
24.0	110.0	34.9	-3.64	-.444	82.81
24.0	115.0	32.6	-3.86	-.507	74.68